DOI: 10.29298/rmcf.v14i78.1330

Research Article

Ecuaciones de razón de volumen para Pinus oocarpa Schiede ex Schltdl. del estado de Nayarit, México

Ratio volume equations of Pinus oocarpa Schiede ex Schltdl. from Nayarit State, Mexico

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Fecha de recepción/Reception date: 30 de diciembre de 2022
Fecha de aceptación/Acceptance date: 30 de junio del 2023

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Abstract
Ratio volume equations are precise mathematical alternatives to estimate merchantable volume of tree species. The objective of the present study was to evaluate the goodness of fit of three commercial volume models associated to height ratio and three taper models to conform a commercial volume-taper equations system of *Pinus oocarpa* growing at the state of *Nayarit*, Mexico. To carry out this study, 76 trees were selected for models fitting and 20 for validation over the entire study area. The models were fitted applying seemingly unrelated regression (*SUR*) in the statistic software SAR 9.2. The goodness of fit of the models was evaluated throughout the comparison of the Coefficient of determination, Root mean square error, Coefficient of variation and Akaike Information Criterium; meanwhile, it was considering the mean bias, absolute mean error, aggregate difference in percentage, Coefficient of determination, Root mean square error and the value of the slope parameter of a lineal regression model for equations validation. The Coefficient of determination and Root mean square error of the best commercial volume model were 0.9727 and 0.0651, and for taper models were 0.9579 and 2.7797, respectively. The validation process allowed to select the commercial volume and taper equations system S2 as the best to estimate volume and diameter at any stem height of *P. oocarpa*.
Key words: Tree taper, *Pinus oocarpa* Schiede ex Schltdl., Schumacher-Hall, commercial volume, ratio volume, total volume.

Resumen

Las ecuaciones de razón de volumen son una opción viable para estimar con precisión el volumen comercial maderable de las especies forestales. El objetivo del presente estudio fue evaluar el ajuste de tres modelos de volumen comercial asociados a la razón de la altura y tres de ahusamiento para conformar un sistema de ecuaciones de volumen comercial-ahusamiento para *Pinus oocarpa* en el estado de Nayarit. Los datos provienen de la medición de 76 árboles para ajustar los modelos y 20 para validarlos. El ajuste se hizo con *PROC MODEL*, y se aplicaron regresiones aparentemente no relacionadas (*SUR*) en el *software* estadístico *SAS* 9.2. Los estadísticos de ajuste fueron el Coeficiente de Determinación Ajustado, Raíz del Cuadrado Medio del Error, Coeficiente de Variación y Criterio de Información de *Akaike*; los de validación fueron el sesgo promedio, error absoluto promedio, porcentaje de la diferencia agregada, Coeficiente de Determinación, Raíz del Cuadrado Medio del Error y los valores del parámetro de la pendiente de la regresión lineal entre datos observados y estimados. El Coeficiente de Determinación y la Raíz del Cuadrado Medio del Error que resultaron del ajuste del mejor modelo de volumen comercial fueron 0.9727 y 0.0651, mientras que los del mejor modelo de ahusamiento fueron 0.9579 y 2.7797. En conclusión, el proceso de validación permitió seleccionar al sistema de ecuaciones de volumen comercial y ahusamiento S2 como el mejor para estimar el volumen y el diámetro a cualquier altura del fuste para *P. oocarpa*.

Palabras clave: Ahusamiento, *Pinus oocarpa* Schiede ex Schltdl., *Schumacher-Hall*, volumen comercial, volumen de razón, volumen total.

Introduction

One of the primary activities in the valuation of forests is the estimation of the total and commercial volumetric stocks of wood in tree species. It has been accomplished by applying independently adjusted (Demaerschalk, 1972; Burkhart, 1977; Clutter, 1980; Lynch *et al.*, 1992) or simultaneously integrated volume and trade models (Fang *et al.*, 2000; Cruz-Cobos *et al.*, 2008; Crecente-Campo *et al.*, 2009; Corral-Rivas *et al*., 2017; Silva-González *et al.*, 2018; Flores *et al.*, 2021). In the first case, although the adjustment of the regression models may be significant, the estimation of the commercial volume throughout the stem presents inconsistencies, evidenced by the crossing of the curves when estimating commercial volumes of trees of different diameter categories (Burkhart, 1977). For the second case, taper equations have been developed, which are adjusted simultaneously with their respective volume equations to estimate both the total volume and the commercial volume (Fang *et al.*, 2000; Cruz-Cobos *et al*., 2008; Silva-González *et al.*, 2018; Flores *et al.,* 2021).

Another alternative to estimate the commercial volume of timber species is through the use of volume ratio equations integrated to one of total volume (Trincado *et al*., 1997; Zhao and Kane, 2017). Based on the minimum diameter or length of the logs required for processing, the percentage of the commercial volume of individual trees is estimated as the ratio of commercial volume over the total volume (Burkhart, 1977; Barrios *et al*., 2014). Although it is recognized that simultaneously adjusted taper and volume systems result in efficient and precise estimators, volume ratio models, apart from being also precise, have the advantage of avoiding complex integration methods in estimating trade volume (Trincado *et al*., 1997); in addition, they also allow to derive compatible taper equations based on the relative heights (García-Espinoza *et al.*, 2018) from them.

Pinus oocarpa Schiede ex Schltdl. is widely distributed naturally over the *Sierra Madre Oriental*, the *Sierra Madre Occidental* and the Neovolcanic Cross Axis (Fabián-Plesníková *et al*., 2020), and therefore, in the temperate zones typical of the mid-mountain state of *Nayarit*. Its soft, moderately heavy and easy-drying wood is used for heavy construction, general use structures, as well as sleepers, packaging, joinery and carpentry, among other uses (Instituto Nacional de Bosques, 2017).

In order to present mathematical options that allow the assessment of timber products efficiently and accurately to contribute to sustainable forest management, this study aims to evaluate the fit of three commercial volume models, composed of the Schumacher-Hall total volume model (Schumacher y Hall, 1933) and the height ratio, and three tapers to form a system of commercial volume and taper equations for *Pinus oocarpa* in the state of *Nayarit*, Mexico.

Materials and Methods

Study area

This research study was carried out in the *Santa María de Picachos ejido*, *Huajicori* municipality, *Nayarit*, Mexico, located in the physiographic region of the *Sierra Madre Occidental* that crosses the northeastern part of the state. The *ejido* has 34 000 ha, mostly covered by pine and oak mixed species that grow between 1 800 and 2 180 masl (Figure 1). The climate is humid semi-warm of C group, the average annual temperature is 18 °C, and the annual mean rainfall was 1 294 mm. The dominant soils are of the eutric Regosol type, followed by eutric Cambisol and ortic Luvisol (INEGI, 2017).

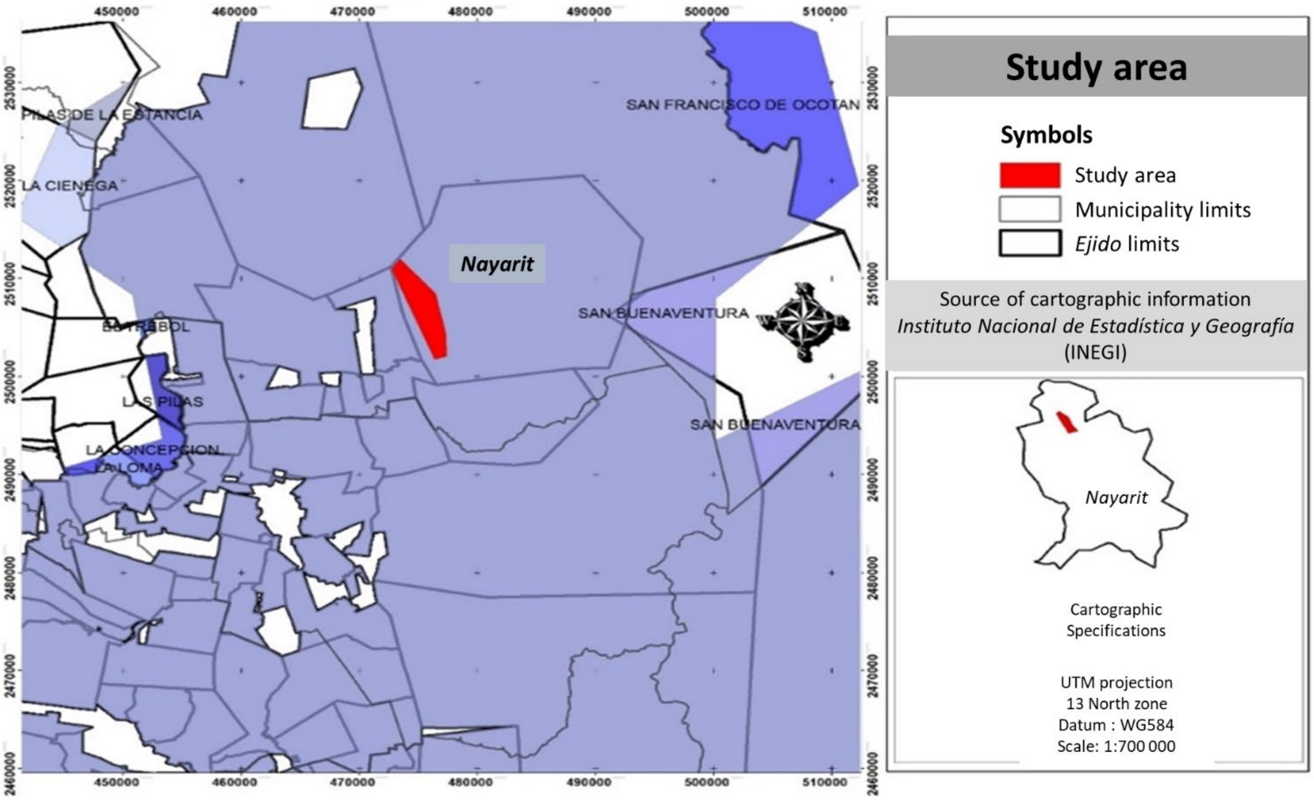

Figure 1. Location of the study area in the *Santa María de Picachos ejido*, *Huajicori* municipality, *Nayarit*, Mexico.

Sampling

The information on taper and volume was obtained from a sample of 96 *P. oocarpa* trees, which were healthy, straight and without damage or physical defects. The sample was representative of all the conditions where the species is distributed within the forest area of the *ejido*. Trees were cut as close to the ground surface as possible; once felled, the diameters of the stem with bark were measured at the height of the cut, 1.3 m from the ground and subsequently at intervals of 2.6 m until reaching the tip of the tree. For this measurement, a 283D Model Forestry Suppliers® diameter tape was used; the lengths of the sections, measured with a FH-8M Model Trupper® flexometer were recorded as the respective heights along the stem.

The database included 1 201 pairs of diameter (d_{ij}) and height (h_{ij}), data including normal diameter (Dn_i) and total height (H_i), where the subscripts $_i$ and $_j$ indicate tree number and any point on the stem, respectively. Of the total pairs of data obtained from 76 trees, 888 were used to fit the models of commercial volume and taper, the complement was used for the validation process.

The volumes of the stump, logs and tip of the stems of each tree were estimated with the geometric equations of the cylinder, Smalian and cone, respectively

(Cancino, 2012). The sum of the volumes of the stump, stem and tip of each tree is equal to the total volume of the stem with bark of each tree.

Volume Ratio, Trade Volume and Taper Models

The volume ratio models $R(p)$ that use as an independent variable the proportion $\left(p = \frac{h}{H}\right)$ of the given heights along the stem (*h*) over the total height (*H*) (Table 1), were selected from Zhao and Kane (2017). Such models have the following properties:

(a) The ratio of the commercial volume to the total volume is equal to zero at the base of the tree when the ratio of the commercial height to the total is equal to zero.
(b) The ratio of the commercial volume to the total volume is equal to one to the total height of the tree when the ratio of the commercial height to the total is equal to one.
(c) The increase in the ratio of the commercial volume to the total volume with respect to the increase in the ratio of the commercial height to the total height would be equal to or greater than zero.
(d) The increase in the ratio of the commercial volume with respect to the increase in the height ratio decreases as the ratio of the commercial height of the tree increases.

Table 1. Simultaneously fitted models of commercial volume and taper for *Pinus oocarpa* Schiede ex Schltdl. in *Santa María de Picachos ejido*, *Huajicori* municipality, *Nayarit*, Mexico.

System	Commercial volume models (*Vh*)	Taper models (*d*)
S1	$a_0 Dn^{\alpha_1} H^{\alpha_2}[1-(1-p)^{\beta_1}]$ (1)	$\sqrt{\frac{\beta_1 a_0 Dn^{\alpha_1} H^{\alpha_2}}{Hk}(1-p)^{\beta_1-1}}$ (4)
S2	$a_0 Dn^{\alpha_1} H^{\alpha_2}[1-(1-p)^{\beta_1}]^{\beta_2}$ (2)	$\sqrt{\frac{a_0 Dn^{\alpha_1} H^{\alpha_2} \beta_1 \beta_2}{kH}[1-(1-p)^{\beta_1}]^{\beta_2-1}(1-p)^{\beta_1-1}}$ (5)
S3	$a_0 Dn^{\alpha_1} H^{\alpha_2}\left[1-(1-p)\frac{\beta_1}{(\beta_1+p)}\right]$ (3)	$\sqrt{\frac{a_0 Dn^{\alpha_1} H^{\alpha_2}(\beta_1^2+\beta_1)}{Hk(\beta_1+p)^2}}$ (6)

S1, S2, S3 = Systems of commercial volume-taper equations; *Dn* = Diameter at 1.3 m height from the ground; *h* = Commercial height of the stem; $p = \frac{h}{H}$; *H* = Total height of the tree; $k = \frac{\pi}{40000}$; π = 3.141592; α_i y β_i = Parameters to estimate.

The commercial volume (*Vh*) models, composed of the Schumacher-Hall total volume model implicit in the volume ratio models, which were adjusted simultaneously with the taper models (*Vh*), were derived and referred by Lynch *et al.* (2017). The simultaneous adjustment of the models allows algebraic compatibility, in such a way that the equations of commercial volume share the same estimators of the parameters with those of taper, and minimize the errors of commercial volume and diameters (Álvarez-González *et al*., 2007; Quiñonez-Barraza *et al*., 2019).

The estimation of commercial volume from the volume ratio requires the application of a total volume equation; in this case, the Schumacher-Hall total volume model was selected, which considers the normal diameter (*Dn*) and the total height of the stem (*H*) as predictor variables:

$$Vt = B_0 Dn^{B_1} H^{B_2} + \varepsilon_i \qquad (7)$$

Where:

Vt = Total volume (m^3)

Dn = Normal diameter (cm)

H = Total height (m)

ε_i = Error term

B_i = Parameters to be estimated

The simultaneous adjustment of the commercial volume models with the taper models was carried out with the PROC MODEL command, in which apparently unrelated regressions (SUR) were applied, in the statistical program SAS 9.2 (Statistical Analysis System, 2004).

Normally, volume estimates present heteroscedasticity problems, which makes it necessary to eliminate their impact. In this study, the heteroscedasticity problem was corrected in the trade volume models using weighted regression. The weighting factor of the models was the reciprocal of $(Dn^2H)^\theta$, where *Dn* and *H* are the allometric variables used in the volume model, θ the parameter that is estimated from the potential regression $e^2 = \rho(Dn^2H)^\theta$ (Parresol, 1999; Zhang *et al.*, 2016; Simental-Cano *et al.*, 2017). The diagnosis of autocorrelation between the errors of the trade volume and taper models was made by applying the Durbin Watson (*DW*) test and the correction with the *CAR(2)* second-order autoregressive model (Zimmerman *et al.*, 2001), using the following structure:

$$e_{ij} = d_1\rho_1{}^{h_{ij}-h_{ij-1}}e_{ij-1} + d_2\rho_2{}^{h_{ij}-h_{ij-2}}e_{ij-2} + \varepsilon ij \qquad (8)$$

Where:

e_{ij} = j^{th} ordinary residual of the i^{th} tree

d_1 = 1 for $j>1$

d_2 = 1 for $j>2$

d_1 = 0 for $j=1$

d_2 = 0 for $j\leq 2$

h_{ij}-h_{ij-1} and h_{ij}-h_{ij-2} = Distances between the observations *j* to *j-1* and *j* to *j-2* within each tree, $h_{ij}>h_{ij}$-1 and $h_{ij}>h_{ij-2}$

ρ_1 and ρ_2 = First and second order autoregressive parameters, respectively

The goodness of fit of the commercial volume-taper systems were evaluated by comparing the adjusted Coefficient of determination (R^2_{adj}), Root mean square of the error (*RMSE*), Akaike's information criterion (*AIC*) and Coefficient of variation (*CV*), while the validation was determined through the average bias (*E*), average absolute error (*EAP*) and the percentage of the added difference (*PDA*) (Diéguez *et al*., 2003; Barrios *et al.*, 2014; García *et al*., 2017). In addition, the relationship between the observed and estimated values was analyzed, using the coefficient of determination (R^2) and the *RMSE* (García *et al*., 2017).

$$R^2_{adj} = \left[1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}\right]\left[\frac{n-1}{n-k-1}\right] \qquad (9)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-k}} \qquad (10)$$

$$AIC = -2\,log(L) + 2k \qquad (11)$$

$$CV = \frac{\sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-k}}}{\bar{y}} \qquad (12)$$

$$E = \frac{\sum(y_i - \hat{y}_i)}{n} \qquad (13)$$

$$EAP = \frac{\sum_{i=1}^{n}|y_i - \hat{y}_i|}{n} \qquad (14)$$

$$PDA = \left[\frac{\sum(y_i - \hat{y}_i)}{n\bar{y}}\right]100 \qquad (15)$$

Where:

y_i = Observed value of the dependent variable

$\hat{y}_i$ = Model predicted value

$\bar{y}$ = Mean value of the dependent variable

n = Number of data used in model fit

k = Number of model parameters

$log(L)$ = Log-likelihood function

Results

The R^2_{adj}, *RMSE*, *AIC* and *CV* statistics derived from the simultaneous adjustment of the compatible models of commercial volume and taper indicated that those that make up the S2 system were the best fit. In turn, by applying the *CAR(2)* second-order autoregressive error structure to the data used to fit the commercial volume models, *DW* values around 1.98 were obtained, while those of taper varied between 1.43 and 1.75, which showed that the correction of the autocorrelation of the errors in the estimation of the commercial volume is fulfilled (Table 2).

Table 2. Goodness-of-fit statistics of the commercial volume (*Vh*) y and taper (*d*) models for *Pinus oocarpa* Schiede ex Schltdl. in *Santa María de Picachos ejido*, *Huajicori* municipality, *Nayarit*, Mexico.

System	Model	R^2_{adj}	*RMSE*	*AIC*	*CV*	*DW*
S1	*Vh*	0.9702	0.0687	836.1585	15.1914	1.9794
	d	0.9319	3.7485	-1 666.7140	11.9578	1.4411
S2	*Vh*	0.9727	0.0651	-1 734.9126	15.1408	1.9805
	d	0.9579	2.7797	614.3040	8.3741	1.4299
S3	*Vh*	0.8187	0.1694	-1 101.5739	32.8997	1.9830

d	0.6412	8.6036	1 356.2510	31.4545	1.7540

R^2_{adj} = Adjusted coefficient of determination; *RMSE* = Root mean square of the error; *AIC* = Akaike information criterion; *CV* = Coefficient of variation; *DW* = Value of the Durbin-Watson statistic.

Based on the significance level of 0.05, all the estimators of the parameters of the fitted models are highly significant (*Pr*<0.0001) (Table 3); therefore, they are reliable in predicting commercial volume and tree taper of *P. oocarpa.*

Table 3. Parameter estimators and associated statistics of the adjusted commercial volume and taper systems in *Pinus oocarpa* Schiede ex Schltdl. in *Santa María de Picachos ejido*, *Huajicori* municipality, *Nayarit*, Mexico.

System	Parameters	Estimation	Standard error	*t*-value	*P<t*
S1	a_0	0.000045	6.471×10^{-6}	6.92	<0.0001
	a_1	1.534708	0.0279	55.09	<0.0001
	a_2	1.499066	0.0544	27.55	<0.0001
	β_1	2.771476	0.0692	40.08	<0.0001
S2	a_0	0.000044	6.568×10^{-6}	6.72	<0.0001
	a_1	1.550813	0.0287	54.02	<0.0001
	a_2	1.483622	0.0561	26.44	<0.0001
	β_1	1.824767	0.0591	30.86	<0.0001
	β_2	0.834866	0.0137	61.10	<0.0001
S3	a_0	0.000036	5.503×10^{-6}	-6.61	<0.0001

a_1	1.535280	0.0293	52.34	<0.0001
a_2	1.567810	0.0568	27.61	<0.0001
β_1	-1.4×10^{62}	2.66×10^{-7}	-53×10^{137}	<0.0001

The validation of the adjusted commercial volume and taper systems indicates that S2 presents the best statistics. The average bias is close to zero, while the average absolute bias, the percentage in accumulated difference and the *RMSE*, present the lowest values, as well as the highest R^2. In addition to the good adjustment of the models of commercial volume and the taper that make up the S2 system, these models are parsimonious and, therefore, easier to apply than others with a greater number of parameters (Table 4).

Table 4. Statistics of the validation process of the commercial volume (*Vh*) and taper (*d*) models adjusted for *Pinus oocarpa* Schiede ex Schltdl. in *Santa María de Picachos ejido*, *Huajicori* municipality, *Nayarit*, Mexico.

System	Model	*E*	*EAP*	*PDA*	R^2	*RMSE*	β_1
S1	*Vh*	-0.0299	0.0496	16.29	0.9761	0.0614	0.985
	d	2.1253	2.9145	12.15	0.9546	3.0593	0.977
S2	*Vh*	-0.0017	0.0397	13.13	0.9773	0.0599	1.036
	d	1.4181	1.8153	6.78	0.9751	2.1336	1.062
S3	*Vh*	0.1035	0.1127	30.34	0.9057	0.1222	1.166
	d	2.8761	5.4071	18.89	0.8098	5.9083	1.036

E = Average absolute bias; *EAP* = Average absolute error; *PDA* = Percentage in accumulated difference; R^2 = Coefficient of determination; *RMSE* = Root mean square error; β_1 = Value of the slope.

The relationships between the observed volume against the estimate and between the observed diameter against the estimate of the S2 system show a linear trend with values of the slope β_1 of 1.036 and 1.062, respectively, which are very close to unity, which confirms that the volume and taper models that make up S2 have a good fit in their predictions (Barrios *et al.*, 2014).

Prior to fitting the commercial volume and taper models, the Schumacher-Hall total volume model was independently fitted to corroborate its efficiency in estimating total volume in *P. oocarpa*. The R^2 and *RMSE* statistics in Table 5 show that this model presents a good fit. In turn, the estimated value of the slope of the linear regression between the observed and the estimated volume (β_1=1.024), as well as the average bias at the tree level (0.00117) applied for validation confirm the good precision of the adjustment of the Schumacher-Hall model.

Table 5. Estimators of the parameters of the total Schumacher-Hall volume model and of the slope parameter of the linear regression applied in the validation in *Pinus oocarpa* Schiede ex Schltdl.

Model	Parameters	Estimation	Standard error	*t*-value	*P<t*
Adjusted from Schumacher-Hall $V = B_0 Dn^{B_1} H^{B_2}$	B_0	0.000065	2.18×10^{-5}	3.002	<0.003
	B_1	1.647	0.0635	25.920	<0.001
	B_2	1.190	0.112	10.564	<0.0001
Linear regression for validation $V_{obs} = B_1 V$	B_1	1.024	0.0136	75.209	<0.0001

V = Total volume estimated with the Schumacher-Hall equation; V_{obs} = Calculated volume with field data; *Dn* = Normal diameter of the stem at 1.30 m height; *H* = Total height of the stem; B_i = Parameter estimators.

Discussion

The decision to fit and apply the Schumacher-Hall volume model to the volume ratio models was made because this model has been successfully fitted to a high diversity of species and regions of Mexico. As examples, Corral-Rivas and Návar-Chaidez (2009), Tapia and Návar (2011), Ramos-Uvilla *et al.* (2014) and Hernández-Ramos *et al.* (2021). Furthermore, in recent years, this volume model has been extensively fitted simultaneously with taper models to estimate the commercial volume of several softwood and broadleaf species by Hernández-Ramos *et al.* (2017), Özçelik and Cao (2017), García-Espinoza *et al.* (2018), Zhao *et al.* (2018) and Hernández-Ramos *et al.* (2021). The analysis of the residuals resulting from the adjustment of the Schumacher-Hall volume model shows that, by including the weighting of the errors, the variance was partially corrected. According to Hernández-Ramos *et al.* (2018), avoiding weighting the errors when fitting the volume models results in an increase in them as the dependent variable increases.

When analyzing the parameter estimators of the implicit volume ratio models in the commercial volume models of the S1 and S2 systems, it is observed that they are within the specifications of Zhao and Kane (2017), who mention that the estimator of the parameter β_1 of the implicit ratio model in the S1 commercial volume equation must be greater than one (β_1=2.771476), and that parameter β_1 associated with the ratio model of the equation to estimate the S2 commercial volume must also be greater than one (β_1=1.824767), while that of β_2 must be between zero and one (β_2=0.834866).

In general, the S2 system showed the highest precision both in estimating trade volume and taper. The validation statistics of the models that make up S2, as well as the linear trend that forms the relationship between the observed data against the estimates, prove that the estimators of the S2 system parameters are efficient (Rachid *et al.*, 2014).

In turn, the statistics reported by Alemdag (1988), when deriving and adjusting several models of volume ratio to *Pinus resinosa* Aiton and *Acer saccharum* Marshall, as well as those of Hernández-Ramos *et al.* (2018), by adjusting various volume ratio equations that are a function of the proportion of diameter at different heights against the normal diameter $\left(\frac{d}{Dn}\right)$ and of the height at different sections of the stem against total height $\left(\frac{h}{H}\right)$ to estimate the commercial volume of *Swietenia macrophylla* King, indicate that the ratio models that are a function of the height proportion are more accurate than those that are a function of the diameter proportion.

In the case of this study, the commercial volume models that have implicit volume ratio models that use the proportion of height, show a high precision in the estimation of commercial volumes.

Garcia-Espinoza *et al.* (2018), when adjusting the Schumacher total volume model with six ratio models, whose independent variable was the proportion of heights $\left(\frac{h}{H}\right)$ for *Pinus pseudostrobus* Lindl., considered as in this study, that both the commercial volume model and the taper model that make up S2 had good adjustments. The R^2 reported by the aforementioned authors were 0.998 and 0.982, the *RMSE* equal to 0.028 and 1.722 for commercial volume and taper, respectively, statistics slightly higher than those estimated in the present study for *P. oocarpa*.

In turn, Zhao and Kane (2017), by adjusting eight ratio equations that comply with the four relative accumulation properties of the stems to estimate the accumulated volume of the *Pinus taeda* L. stem, also determined that S2 was the best, followed by S1.

Quiñonez-Barraza *et al.* (2019) revealed that, of 11 systems adjusted to five pine species, S2 was considered the most parsimonious for presenting fewer than six parameters, which is why they also selected the S2 system as the second best for predicting both commercial volume and the taper. Because the expression of the commercial height (h) is undefined for the taper model of the S2 system, it can be estimated through iterations using numerical methods designed for it (Lynch *et al.,* 2017).

Conclusions

The analysis on the simultaneous and compatible adjustment of commercial volume and taper systems, showed that the volume ratio models are a reliable option to estimate the commercial volume of *Pinus oocarpa* trees. According to the adjustment statistics and the validation process, the commercial volume and taper models of the S2 system allow to reliably determine the commercial volume, the total volume, as well as the stem profile in specimens of *P. oocarpa* in the study area.

Acknowledgements

The authors thank the owners of the *Santa María de Pichachos ejido* for allowing the information to be collected within their land, as well as the *Tecnológico Nacional de México* through the *Instituto Tecnológico de El Salto* for the financial facilities granted to obtain the field information.

Conflict of interest

The authors declare that they have no conflict of interest regarding the publication of this document.

Contribution by author

Francisco Javier Hernández and Luis Alberto Simental Serrano: design, data collection and analysis and writing of the manuscript; José Ciro Hernández Díaz, Christian A. Wehenkel, José Ángel Prieto Ruíz and Juan Abel Nájera Luna: data analysis, discussion process and review of the manuscript.

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