



## Modelos para determinar las relaciones alométricas en *Juniperus deppeana* Steud. en el estado de Tlaxcala

## Models for determining allometric relationships in *Juniperus deppeana* Steud. in the state of Tlaxcala

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### Abstract

Quantifying the stocking of trees by means of allometric relationships within a stand favors better forest management, timber harvesting and helps to estimate losses due to illegal logging. The objective of this study was to represent in a quantitative way the allometric relationships between the variables of forest interest for *Juniperus deppeana*, a species of restricted use. Several allometric models were fitted with 1 096 pairs of data for stump (*sd*), normal (*nd*), and crown (*cd*) diameters, total height (*Th*), and volume (*V*) of trees of two growth conditions in natural forests of northern and western Tlaxcala State, Mexico. An average statistical improvement of 16.63 % in the explanation of sampling variability and a reduction of 18.53 % in bias were obtained by including the site as a random effect in the mixed-effects modeling approach. Validation of the estimates with data that are independent of the adjustment showed no significant differences. Consistently, the estimates of *cd*, *Th*, or *V* as a function of *sd* were conservative compared to when *nd* is used as an explanatory variable. Models  $nd = 2.5413 + 0.6778 \times sd$ ,  $R^2=0.8187$ ;  $cd = 0.4513 \times nd^{0.7733}$ ,

$R^2=0.8195$ ;  $Th = 1.3382 \times nd^{0.5039}$ ,  $R^2=0.6281$ , and  $V = 0.0003 \times nd^{2.1005}$ ,  $R^2=0.9563$  proved to be

reliable for reconstructing the mensuration characteristics of trees in stands affected by illegal logging activities, and for accurately quantifying the actual stock of this species' forests, they can be a reliable option for *Juniperus deppeana* forest management plans.

**Keywords:** Allometric equations, restricted harvesting species, post-harvest assessment, forest management, mixed effects model, juniper.

#### Resumen

Cuantificar las existencias del arbolado por medio de las relaciones alométricas dentro de un rodal favorece una mejor gestión forestal, aprovechamiento maderable y ayuda a estimar las pérdidas por cortas clandestinas. El objetivo del presente estudio fue representar de manera cuantitativa las relaciones alométricas entre las variables de interés forestal para *Juniperus deppeana*, especie de aprovechamiento restringido. Se ajustaron distintos modelos alométricos con 1 096 pares de datos de los diámetros de tocón (*dt*), normal (*dn*) y de copa (*dc*), altura total (*At*) y volumen (*V*) de árboles de dos condiciones de crecimiento en bosques naturales del norte y poniente de Tlaxcala, México. Se obtuvo una mejora estadística promedio del 16.63 % en la explicación de la variabilidad muestral y una reducción de 18.53 % en el sesgo al incluir el sitio como efecto aleatorio en el enfoque de modelos de efectos mixtos. La validación de las estimaciones con datos independientes al ajuste no mostró diferencias significativas. De manera consistente, las estimaciones de *dc*, *At* o *V* en función del *dt* fueron conservadoras en comparación a cuando se utiliza como variable explicativa al *dn*. Los modelos

$$\text{dn} = 2.5413 + 0.6778 \times dt, R^2=0.8187; dc = 0.4513 \times dn^{0.7733}, R^2=0.8195; At = 1.3382 \times dn^{0.5039},$$

$R^2=0.6281$  y  $V = 0.0003 \times dn^{2.1005}$ ,  $R^2=0.9563$  evidenciaron ser confiables para reconstruir las características dasométricas del arbolado dentro de los rodales afectados por actividades de clandestinaje, y para cuantificar las existencias reales de los bosques de esta especie de forma precisa. Debido a lo anterior, pueden ser una alternativa confiable para la elaboración de planes de manejo forestal para *Juniperus deppeana*.

**Palabras clave:** Ecuaciones alométricas, especie de aprovechamiento restringido, evaluación posclandestinaje, manejo forestal, modelo de efectos mixtos, sabino.

## Introduction

*Juniperus* is the second most diverse genus of the conifer group in the world, second only to *Pinus*. Mexico is home to 16 species, of which *Juniperus deppeana* Steud. and *Juniperus flaccida* Schldtl. are the most widely distributed (Farjon, 2005). *J. deppeana* forms coves in different geographic regions of Mexico. Particularly in the state of *Tlaxcala*, it covers approximately 12 711 ha; it is located in small and fragmented populations, to such extent that only patches of original vegetation can be observed (Isla et al., 2008; Herrerías and Nieto de Pascual, 2020). In the central-western region (study area), only 24.6 % of the forested area is subject to forest management based on technical criteria, which, together with the lack of monitoring, increases forest degradation, mainly due to land clearing for

agricultural and livestock purposes, as well as to the extraction of firewood for fuel (Conafor, 2003). For this reason, it is necessary to know the dimensions of the subtracted trees (normal diameter, height, volume, biomass, carbon, among others), which is difficult if there are no models for estimating the dasometric variables of the trees such as normal diameter ( $nd$ , cm), total height ( $Th$ , m) or volume ( $V$ ,  $m^3$ ) from the stump diameter ( $sd$ , cm) (residual variable). Based on this information, the losses can be quantified, and an appropriate forest management strategy can thus be designed (García-Cuevas *et al.*, 2017).

Within this context, the statistical modeling of the allometric relationships of trees is a useful resource for a correct estimation of their dimensions (Pompa-García *et al.*, 2011; García-Cuevas *et al.*, 2016). These tools are reliable and reduce both the time and resources used for obtaining forest inventory information (Picard *et al.*, 2012).

In general, the dasometric variables have been statistically modeled by fitting regression techniques using the Ordinary Linear Squares (OLS) method (Hernández *et al.*, 2016; García-Cuevas *et al.*, 2017; Guerra-De la Cruz *et al.*, 2019). However, these have some limitations when used with correlated data from the same locality, site or sampling unit, or from repeated measurements of a variable over time (Calama and Montero, 2004; Corral *et al.*, 2019).

The application of Mixed Effects Models (MEM) is an alternative for statistical improvement, which allows the inclusion of covariates that reduce the error and the specific variability by classification level (Baty *et al.*, 2015; Correa and Salazar, 2016; Corral *et al.*, 2019). Therefore, and because there are no models in the region of *Tlaxcala* for estimating the mensuration variables in *Juniperus deppeana* trees, the objective of this study was to represent in a quantitative way the allometric relationships between the variables of forest interest— $sd$ ,  $nd$ ,  $cd$ ,  $Th$ , and  $V$ — for *Juniperus deppeana* trees, through the incorporation of mixed-effects modeling.

## Materials and Methods

### Study area

The study was carried out in two localities of *Tlaxcala*: *Santa María EspaÑita*, in the *EspaÑita* municipality, and *San Pedro Ecatepec ejido*, in the *Atlangatepec* municipality. The *Santa María EspaÑita* forest zone is located in the western part of the state, between 19°30'00" and 19°28'40" N, and 98°26'25" and 98°25'20" W with an average altitude of 2 720 m; *Atlangatepec* is located in the north of the state, between 19°34'00" and 19°32'45" N, and 98°07'50" and 98°08'45" W, at an average altitude of 2 560 m.

The climate is type C(w<sub>2</sub>), with rainfall from July to September, averaging 700 to 1 215 mm; the mean annual temperature is 14.5 °C (García, 2004). The original vegetation is very fragmented and is mainly composed of mixed forests of *J. deppeana*, *Pinus* spp. and *Quercus* spp., in particular *Q. obtusata* Bonpl., *Q. castanea* Née and *Q. frutex* Trel. (INEGI, 2010). The floristic composition is varied, with a strong impact of natural and anthropogenic disturbances, evidenced by the fact that the native vegetation is disturbed or has been displaced by agriculture or livestock (Castañeda, 2015).

### Data collection

Information was collected from 372 *J. deppeana* trees in the *España* area and from 724 trees in *Atlangatepec* (1 096 pairs of data), corresponding to 78 sampling 400 m<sup>2</sup> sites (25 in *España* and 53 in *Atlangatepec*), randomly placed on a surface area of 1 000 ha. Each tree with a normal diameter larger than 7.5 cm (*nd*=1.3 m) was measured for stump diameter (*sd*, cm) and normal diameter (*nd*, cm) with a model 283D Forestry Suppliers, Inc.<sup>®</sup> brand diameter tape, and for crown diameter (*cd*, m) with a 30 m model 47322 ProTape<sup>®</sup> brand fiberglass tape. *Th* measurements were made with a model T3 Vertex III Haglöf<sup>®</sup> Vertex<sup>®</sup> digital hypsometer with ultrasonic transmitter. Subsequently, for each individual, the *V* was calculated with the equation:  $V = 0.00024698 \times nd^{1.6254} \times Th^{0.855}$  ( $R^2=0.9342$ ,  $RMSE=0.2442$ ) (Graciano-Ávila *et al.*, 2019). In addition to and independently of the aforementioned sample, 341 pairs of data were obtained with the same methodology to carry out the validation process of the models.

## Data analysis

In a first analysis with 70 % of the total information, and in order to estimate the initial or seed parameters of each allometric relationship (dependent variable: *nd* and *sd*, and independent variable: *cd*, *Th*, and *V*), the different mathematical structures proposed in the specialized literature were adjusted (Table 1) (Quiñonez *et al.*, 2012; Hernández *et al.*, 2016; Guerra-De la Cruz *et al.*, 2019). This was done with the linear models (*lm*) functions by means of the Ordinary Least-Squares (OLS)

and nonlinear least-squares (NLLS) methods; RStudio® software version 2022.07.2 Build 576 was used for this purpose (R Core Team, 2022).

**Table 1.** Adjusted models to quantify the allometric relationship of the dasometric variables.

No.	Model	Expression	Allometric relationship
1	Linear	$y = a_0 + a_1x$	nd-sd
2	Potential	$y = a_0x^{a_1}$	cd-sd, cd-nd, Th-sd, Th-nd, V-sd, V-nd

$y$  = Dependent variable ( $sd$  = Stump diameter (cm) and  $nd$  = Normal diameter (cm));  $x$  = Independent variable ( $cd$  = Crown diameter (m),  $Th$  = Total height (m), and  $V$  = Stem volume ( $m^3$ ));  $a_0$  and  $a_1$  = Parameters.

Subsequently, in a second statistical approach, the structures of Table 1 were refitted with the *nlme* function by maximum likelihood using MEM in the same version of RStudio® (Pinheiro et al., 2022; R Core Team, 2022). For this purpose, the site effect ( $a_i$ ) was included as an additive covariate (e.g.,  $y = a_0x^{a_1+a_i}$ ) in order to group (in the *Españita* or *Atlangatepec* site) and explain in greater detail the sample variability of the dimensions of the variables of forestry interest, and as a way of correcting the problems of heteroscedasticity that commonly occur in this type of research (Quiñonez-Barraza et al., 2018).

Mixed effects were incorporated individually for each parameter (e.g.,  $y = a_0x^{a_1+a_i}$ ;  $y = (a_0 + a_i) \cdot x^{a_1}$ ) and all its potential combinations (e.g.,  $y = (a_0 + a_i) \cdot x^{(a_1+a_i)}$ ), where  $a_i$  indicates the position of the random effects included in the models (Pinheiro and Bates, 2013; Gałecki and Burzykowski, 2013).

## Fit assessment and analysis validation

Based on the contrast of the allometric relationship fits between OLS and NLLS *versus* the MEM approach, the quality of fit was verified by means of the Coefficient of determination ( $R^2$ ), the Root mean square of the error (RMSE), and the significance level of the estimated parameters ( $\alpha < 0.05$ ). Likewise, the deviations of each estimation by best selected expression were quantified through the average error (Bias) (Corral *et al.*, 2019; Guerra-De la Cruz *et al.*, 2019). In addition, the assumptions of the regression model were checked to ensure that they were met: normality (graphical form), homogeneity of variance (homoscedasticity) (graphical form), and independence of the residuals. It was also verified that all parameters of the selected model were significantly different from zero ( $\alpha < 0.05$ ) (Martínez *et al.*, 2006; Zar, 2010).

In order to perform the validation, it was hypothesized that the estimates obtained with the models are equal to the data of the validation sample ( $H_0$ : null hypothesis), and, as an alternative hypothesis ( $H_a$ ), it was suggested that the information differs between the two models. For this purpose, the Student's t-test was used at a 95 % reliability ( $\alpha < 0.05$ ) to test hypotheses about means in normally distributed populations with information from 341 completely random and independent pairs of data not used for model fitting (Zar, 2010).

## Results

Descriptive statistics indicated that stump diameter ranged from 9.0 cm to 96.3 cm, the normal diameter of 7.5 cm to 88.0 cm with an average of 22.0 cm, and crown diameter from 0.9 m to 15.9 m. For total height the variation recorded was from 1.9 m to 17.0 m, and for the stem volume, from 0.0131 m<sup>3</sup> to 4.0296 m<sup>3</sup> per tree (Table 2).

**Table 2.** Basic statistics of the mensuration variables used for the adjustment and validation of the allometric equations.

<b>Variable/ Statistic</b>	<b>sd</b>	<b>nd</b>	<b>cd</b>	<b>Th</b>	<b>V</b>	<b>sd</b>	<b>nd</b>	<b>cd</b>	<b>Th</b>	<b>V</b>
	Calibration data (70 %)					Validation data (30 %)				
Account		1 096					341			
Minimum	9.0	7.5	0.9	1.9	0.0131	10.0	7.5	1.4	2.5	0.0192
Maximum	96.3	88.0	15.9	17.0	4.0296	92.3	88.0	12.2	14.0	2.2278
Mean	29.0	22.0	4.8	6.2	0.2304	28.9	22.0	4.9	6.2	0.2225
Standard error	0.408	0.325	0.065	0.069	0.009	0.725	0.574	0.116	0.115	0.013
Standard deviation	13.518	10.765	2.154	2.274	0.287	13.382	10.604	2.137	2.121	0.247
Variance	182.725	115.887	4.641	5.173	0.082	179.080	112.440	4.567	4.498	0.061

*sd* = Stump diameter (cm); *nd* = Normal diameter (cm); *cd* = Crown diameter (m);  
*Th* = Total height (m); *V* = Stem volume (m<sup>3</sup>).

OLS fits for the linear model and NLLS fits for the allometric expression were significant for all parameters (*a*<0.05), and the global deviations in the estimates resulted in less than two units (*RMSE* value), with the exception of the *nd-sd* ratio, which registered a value of 5.4 cm. Therefore, the selected equations correctly predict the *nd-sd* and *cd-nd* relationships with an *R*<sup>2</sup>=0.7517 and 0.7741, respectively; on the other hand, for

the prediction of volume with respect to the  $nd$  was the best ratio with an  $R^2=0.9442$  (Table 3). The models for predicting  $Th$  from  $sd$  and  $nd$  had the lowest  $R^2$  ( $<0.50$ ) in both cases (Table 3), this is due to the great variability in heights (Variance 5.173, Table 2) exhibited by the trees in the field.

**Table 3.** Parameters and statistics of allometric models using the linear (OLS) and nonlinear (NLLS) ordinary least squares methods.

Relationship	Model	Parameter	Estimation	Standard error	t value	Pr> t	R <sup>2</sup>	RMSE
nd-sd	1	$a_0$	1.8430	0.4654	3.96	<0.0001	0.7517	5.4003
		$a_1$	0.6920	0.0145	47.74	<0.0001		
Th-sd	2	$a_0$	1.1085	0.0901	12.30	<0.0001	0.4010	1.8111
		$a_1$	0.5197	0.0232	22.43	<0.0001		
cd-sd	1	$a_0$	0.4130	0.0333	12.41	<0.0001	0.5948	1.3761
		$a_1$	0.7363	0.0224	32.81	<0.0001		
V-sd	2	$a_0$	0.0001	0.0000	4.69	<0.0001	0.6729	0.1735
		$a_1$	2.2400	0.0527	42.53	<0.0001		
Th-nd	2	$a_0$	1.2010	0.0804	14.93	<0.0001	0.4746	1.6961
		$a_1$	0.5416	0.0206	26.30	<0.0001		
cd-nd	1	$a_0$	0.4151	0.0218	19.08	<0.0001	0.7741	1.0274
		$a_1$	0.8008	0.0156	51.35	<0.0001		
V-nd	2	$a_0$	0.0002	0.0000	16.18	<0.0001	0.9442	0.0717
		$a_1$	2.1540	0.0158	136.48	<0.0001		

$nd$  = Normal diameter (cm);  $sd$  = Stump diameter (cm);  $Th$  = Total height (m);  $cd$  = Crown diameter (m);  $V$  = Stem volume ( $m^3$ );  $R^2$  = Coefficient of determination;  
 $RMSE$  = Root mean square error.

When applying the MEM technique, better statistical fit values were observed for all relationships, since the random effect of site was included in an additive way, as a variable that groups individuals in one of the two locations (*Españita, Atlangatepec*), which ranged from 1.3 % in explaining the sample variability of  $V$  as a function of  $nd$ , to 42.3 % in the  $Th-sd$  relationship. Furthermore, the  $RMSE$  value decreased by 54.4 % for  $nd-sd$ , and by 7.4 % when  $V$  was estimated as a function of the stump dimensions (Table 4). The intervals of the estimated parameters, fixed and random effects fit value, and the variance-covariance matrix (vcov) of each allometric relationship are shown in Table 5.

**Table 4.** Parameters and statistics of allometric models using the mixed effects modeling (MEM) approach.

Relationship	Parameter	Estimation	Standard error	t value	R <sup>2</sup>	RMSE	Bias	>%-R <sup>2</sup>	>%-RMSE
<i>nd-sd</i>	$a_0^*$	2.5413	0.604	4.21	0.8187	2.4624	<0.01	8.9	54.4
	$a_1$	0.6778	0.016	43.52					
<i>Th-sd</i>	$a_0$	1.2512	0.099	12.58	0.5706	1.5343	0.02	42.3	15.3
	$a_1^*$	0.4819	0.024	19.79					
<i>cd-sd</i>	$a_0$	0.4490	0.036	12.47	0.7062	1.1727	0.01	18.7	14.8
	$a_1^*$	0.7124	0.024	29.44					
<i>V-sd</i>	$a_0$	0.0002	0.000	8.76	0.7199	0.1606	<0.01	7.0	7.4
	$a_1^*$	1.9788	0.036	55.23					
<i>Th-nd</i>	$a_0$	1.3382	0.088	15.13	0.6281	1.4280	0.01	32.3	15.8
	$a_1^*$	0.5039	0.022	22.79					
<i>cd-nd</i>	$a_0$	0.4513	0.026	17.46	0.8195	0.9192	<0.01	5.9	10.5
	$a_1^*$	0.7733	0.018	42.75					
<i>V-nd</i>	$a_0^*$	0.0003	0.000	17.28	0.9563	0.0634	0.00	1.3	11.5

$a_0$  2.1005 0.018 119.01

$sd$  = Stump diameter (cm);  $nd$  = Normal diameter (cm);  $cd$  = Crown diameter (m);  
 $Th$  = Total height (m);  $V$  = Stem volume ( $m^3$ );  $R^2$  = Coefficient of determination;  
 $RMSE$  = Root mean square error;  $>\%R^2$  = Percentage improvement of the fit in the  
value of the Coefficient of determination;  $>\%-RMSE$  = Percentage improvement of  
the fit to the root mean square error value.

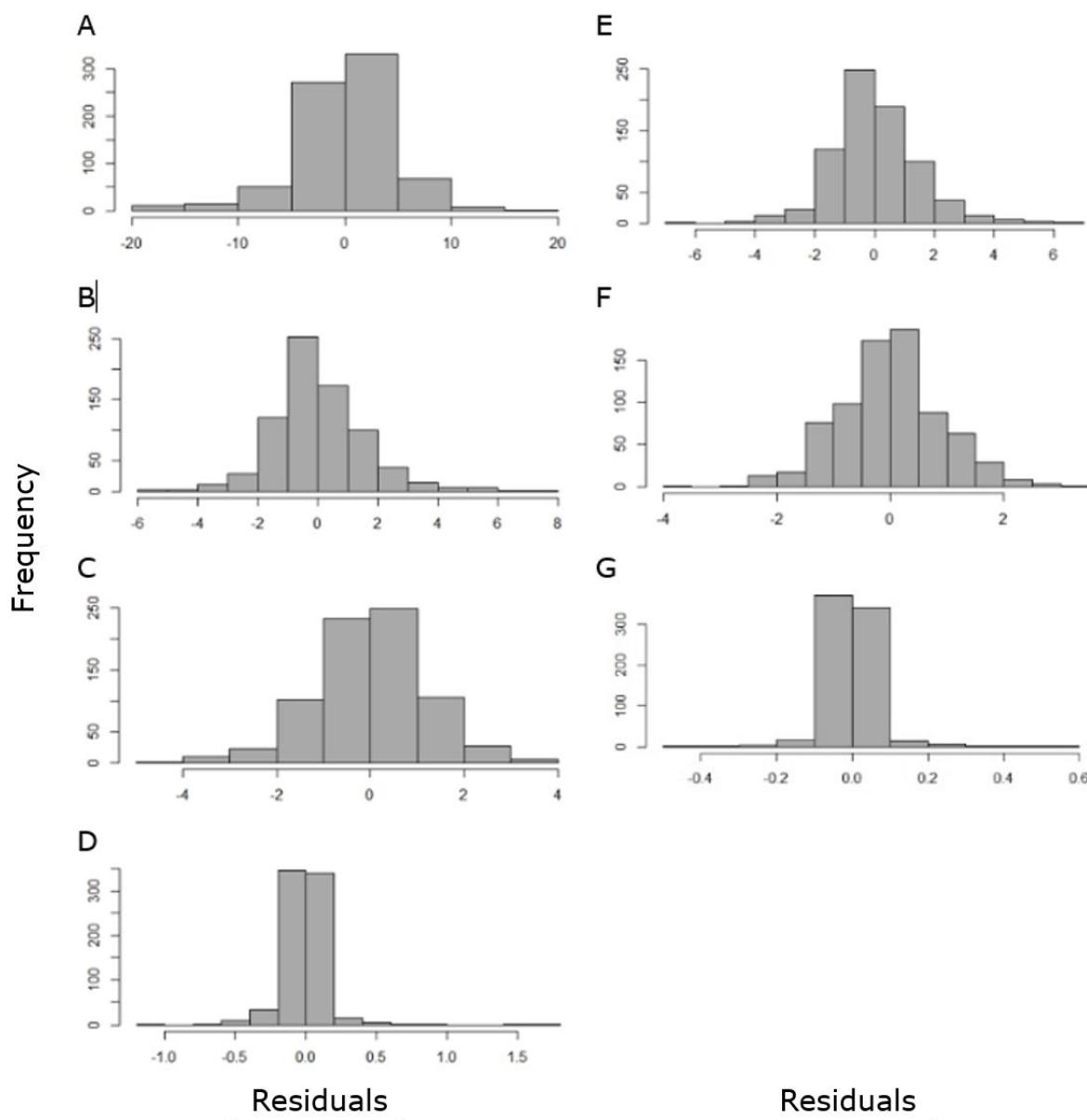
**Table 5.** Intervals of the estimated parameters, fixed and random effects fit value, and variance-covariance matrix (vcov) of each allometric relationship.

Relationship	Parameter	Lower limit	Estimated value	Upper limit	vcov matrix	
					$a_0$	$a_1$
$nd-sd$	$a_0$	1.3573	2.5413	3.7252	0.3636	-0.0076
	$a_1$	0.6473	0.6778	0.7083	-0.0076	0.0002
	$SD(a_0)$	1.4622	2.1008	3.0182	Residual:	5.0955
	G-SE	4.8309	5.0955	5.3745		
$Th-sd$	$a_0$	1.0562	1.2512	1.4461	0.0099	-0.0023
	$a_1$	0.4342	0.4819	0.5297	-0.0023	0.0006
	$SD(a_1)$	0.0345	0.0439	0.0560	Residual:	0.3151
	G-SE	0.2224	0.3151	0.4466		
$cd-sd$	$a_0$	0.6760	0.8691	1.0623		
	$a_1$	0.3784	0.4490	0.5196	0.0013	-0.0008
	$SD(a_1)$	0.6650	0.7124	0.7599	-0.0008	0.0006
	G-SE	0.0324	0.0411	0.0523	Residual:	0.3813
$V-sd$	$a_0$	0.5734	0.7222	0.8710		
	$a_1$	0.0002	0.0002	0.0003	7.55E-10	-9.49E-07
	$SD(a_1)$	1.9086	1.9788	2.0491	-9.49E-07	1.28E-03
	G-SE					

	$SD(a_1)$	0.0448	0.0582	0.0756	Residual:	0.4722
	G-SE	0.4199	0.4722	0.5309		
	$Vvf(p)$	1.0685	1.1238	1.1791		
<i>Th-nd</i>	$a_0$	1.1648	1.3382	1.5117	0.0078	-0.0019
	$a_1$	0.4606	0.5039	0.5473	-0.0019	0.0005
	$SD(a_1)$	0.0343	0.0437	0.0558	Residual:	0.3678
<i>cd-nd</i>	G-SE	0.2628	0.3678	0.5149		
	$Vvf(p)$	0.5630	0.7497	0.9365		
	$a_0$	0.4006	0.4513	0.5020	0.0007	-0.0005
<i>V-nd</i>	$a_1$	0.7378	0.7733	0.8087	-0.0005	0.0003
	$SD(a_1)$	0.0206	0.0273	0.0362	Residual:	0.5715
	G-SE	0.4638	0.5715	0.7043		
	$Vvf(p)$	0.1900	0.3248	0.4595		
	$a_0$	0.0002	0.0003	0.0003	2.55E-10	-2.68E-07
	$a_1$	2.0659	2.1005	2.1351	-2.68E-07	3.11E-04
	$SD(a_0)$	2.34E-05	3.11E-05	4.13E-05	Residual:	0.1707
	G-SE	0.1517	0.1707	0.1921		
	$Vvf(p)$	0.8573	0.9108	0.9643		

*sd* = Stump diameter (cm); *nd* = Normal diameter (cm); *cd* = Crown diameter (m);  
*Th* = Total height (m); *V* = Stem volume ( $m^3$ ); *SD* = Standard deviation of the effect parameter; G-SE = Within-group standard error; *Vvf(p)* = Value of the variance function of power type.

No issues of non-compliance with the regression assumptions were observed for the fit in any of the proposed allometric relationships when the MEM was used, because the frequency distribution of the residuals is Gaussian (bell-shaped) for all variables (Figure 1), and the dispersion of the residuals showed a random trend (Figure 2).

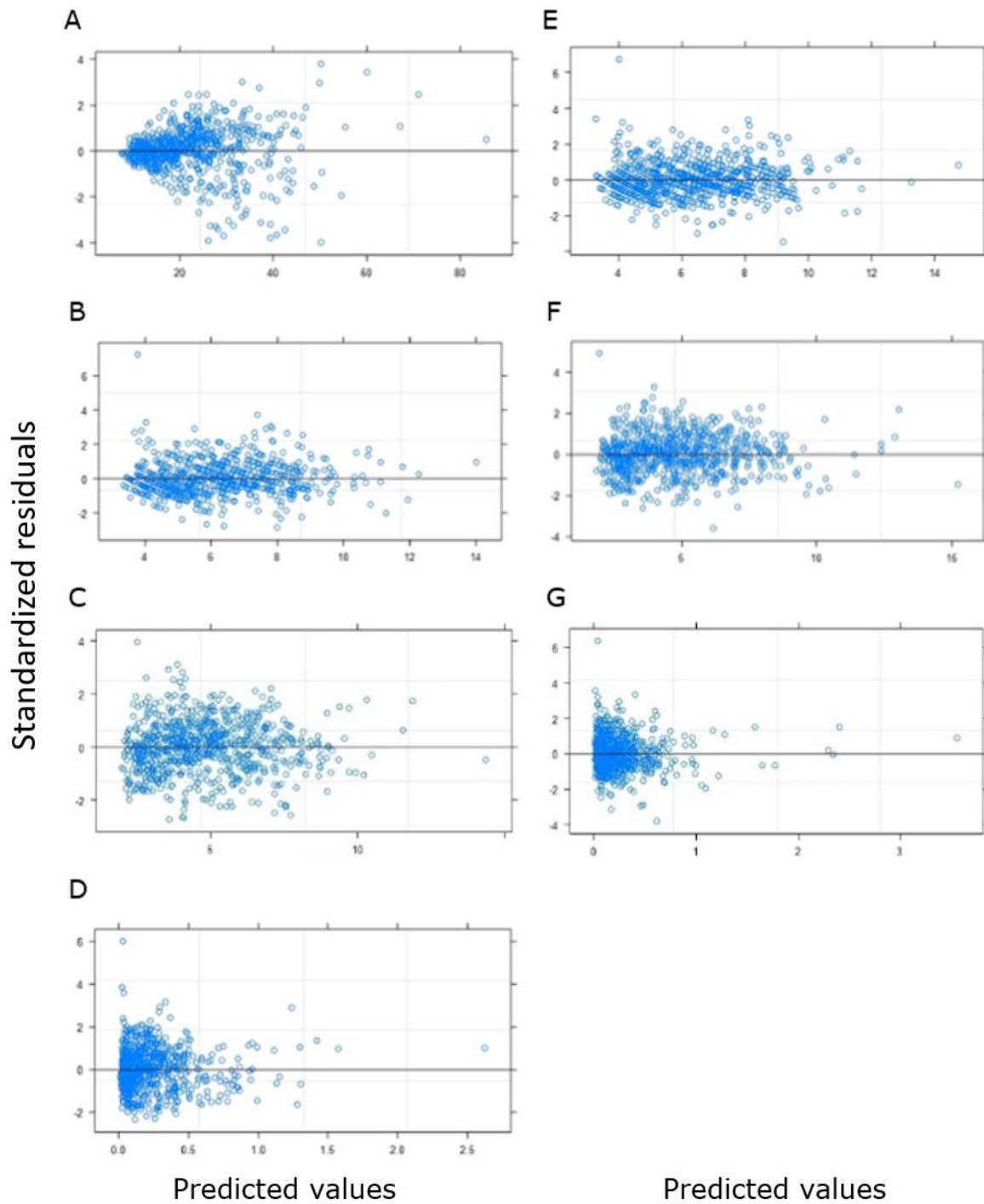


A =  $nd-sd$ ; B =  $Th-sd$ ; C =  $cd-sd$ ; D =  $V-sd$ ; E =  $Th-nd$ ; F =  $cd-nd$ ; G =  $V-nd$ .

$sd$  = Stump diameter (cm);  $nd$  = Normal diameter (cm);  $cd$  = Crown diameter (m);

$Th$  = Total height (m);  $V$  = Stem volume ( $m^3$ ).

**Figure 1.** Graphical normality test for the proposed models in each allometric relationship.



A =  $nd-sd$ ; B =  $Th-sd$ ; C =  $cd-sd$ ; D =  $V-sd$ ; E =  $Th-nd$ ; F =  $cd-nd$ ; G =  $V-nd$ .

$sd$  = Stump diameter (cm);  $nd$  = Normal diameter (cm);  $cd$  = Crown diameter (m);  
 $Th$  = Total height (m);  $V$  = Stem volume ( $m^3$ ).

**Figure 2.** Graphical homoscedasticity test for the proposed models in each allometric relationship.

The validation of the estimates made with the MEM-adjusted models for the various allometric relationships showed no significant differences ( $p < 0.05$ ) in any case. Therefore,  $H_0$  for equality of means is accepted, while  $H_a$  is rejected (Table 6).

**Table 6.** Validation test between the estimates of each allometric relationship and observed data of the independent sample.

Validation	Independent variable	t value	p value
<i>nd</i>	Stump diameter ( <i>sd</i> , cm)	-0.2332	0.816
<i>cd</i>		0.1356	0.981
<i>Th</i>		-0.0476	0.962
<i>V</i>		-0.1996	0.842
<i>cd</i>	Normal diameter ( <i>nd</i> , cm)	0.0475	0.962
<i>Th</i>		-0.0835	0.934
<i>V</i>		-0.4201	0.675

*cd* = Crown diameter (m); *Th* = Total height (m); *V* = Stem volume ( $\text{m}^3$ ).

In order to exemplify the application of the models, a clandestine logging area of one hectare with a density of 190 individuals was assumed, with *J. deppeana* trees whose *sd* averages 29 cm. Therefore, when applying model 1 ( $y = a_0 + a_1x$ : linear) to the *nd-sd* relationship, we have:  $nd = 2.5413 + 0.6778 \times 29 = 22.2 \text{ cm}$ .

Subsequently, the *cd-nd*, *Th-nd*, and *V-nd* ratios were calculated with the structure of model 2 ( $y = a_0x^{a_1}$ : potential):  $cd = 0.4513 \times 22.2^{0.7733} = 4.96 \text{ m}$ ;  $Th = 1.3382 \times 22.2^{0.5039} = 6.38 \text{ m}$  and  $V = 0.0003 \times 22.2^{2.1005} = 0.2019 \text{ m}^3$ .

This procedure allows us to determine the dasometric characteristics of the trees within the affected stand. By multiplying the  $V$  by the density, it is possible to quantify the surface area subjected to clandestine felling, whose value would be 38.3609 m<sup>3</sup>. The same procedure can be performed by considering the  $sd$ .

## Discussion

In general, the traditional approach provided acceptable results for the different allometric relationships, as in other related works (Quiñonez et al., 2012; Hernández et al., 2016; Guerra-De la Cruz et al., 2019). However, the inclusion of the site within the MEM approach improved the average explanation of sampling variability by 16.63 % and reduced model deviations by 18.53 %. This situation is attributed to the fact that the variability of the analyzed information is grouped by site (Castedo et al., 2006; Corral et al., 2019).

The proposed volume model ( $R^2=0.9563$ ) does not explain the sample variability as well as the model utilized by Buendía-Rodríguez et al. (2022) ( $R^2=0.971$ ), which is also potential; however, these authors only adjusted a homogeneous population of *J. deppeana*. The resulting gain was improved with the implementation of the MEM for the  $V-sd$  (7 %) and  $V-nd$  (1.3 %) ratios, with which higher increases were achieved than those quoted by Guangyi et al. (2015) of 1.3-2 %, when estimating the volume of *Cunninghamia lanceolata* (Lamb.) Hook trees.

The  $nd-sd$ ,  $cd-sd$  and  $cd-nd$  models had conservative gains of 8.9, 18.7 and 5.9 %, respectively, with  $R^2$  values of 0.8187 ( $nd-sd$ ), 0.7062 ( $cd-sd$ ), and 0.8195 ( $cd-nd$ ),

which are lower than those recorded by Pompa-García *et al.* (2011) in *Pinus durangensis* Martínez ( $R^2=0.96$ ) for *nd-sd*.

In a similar way, the models fitted by MEM (*Th-sd*, *Th-nd*) exhibited a better fit (0.5706 and 0.6281) than those documented by Buendía-Rodríguez *et al.* (2022), who used a traditional OLS fit ( $R^2=0.427$ ).

Guerra-De la Cruz *et al.* (2023) report that the incorporation of MEM in order to fit *Th-nd* models, as well as of the covariate sub-watershed as a grouping factor in *Abies religiosa* (Kunth) Schltdl. & Cham., results in gains of 12.19 % ( $R^2$ ), while, in this study, 32.3 % gain was achieved, and a better result was obtained with the grouping factor (site) for *J. deppeana* forests. In this regard, Salas-Eljatib *et al.* (2019) indicate that, by incorporating the effect of the covariate with the MEM approach, the variance is reduced and rendered homogeneous.

*Juniperus deppeana* requires forest management based on the use of models according to the mensuration characteristics of each region, since it is a species with great interspecific variation (Flores *et al.*, 2018), which in turn helps maintain biodiversity, as other taxa are associated with it (Maxted, 2013), such as *Quercus potosina* Trel. and *Pinus cembroides* Zucc. (Díaz-Núñez *et al.*, 2016). Therefore, it is a conifer with high ecological and scientific potential.

## Conclusions

Allometric models fitted using the mixed-effects approach satisfactorily explain the behavior of the analyzed mensuration variables

$$nd = 2.5413 + 0.6778 \times dt \quad R^2=0.8187$$

$$cd = 0.4513 \times dn^{0.7733} \quad R^2=0.8195$$

$$Th = 1.3382 \times dn^{0.5039} \quad R^2=0.6281$$

$$V = 0.0003 \times dn^{2.1005} \quad R^2=0.9563$$

In addition, modeling using the MEM technique improves the fit with respect to the traditional linear Ordinary Least Squares (OLS) and nonlinear ordinary least squares (NLLS) techniques.

The results prove to be reliable for reconstructing the dasometric characteristics of trees within stands affected by illegal logging activities and for accurately quantifying the actual stocking of *J. deppeana* forests; therefore, they are a reliable alternative for the preparation of forest management plans in *Tlaxcala* and neighboring states.

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## Conflict of interest

Enrique Buendía Rodríguez declares not to have participated in the editorial process of the manuscript.

### **Contribution by author**

Eulogio Flores Ayala: planning and development of the research, data collection; Tomás Pineda Ojeda: data collection and drafting of the manuscript; Jonathan Hernández Ramos: statistical analysis, drafting and revision of the manuscript; Enrique Buendía Rodríguez: statistical analysis, drafting, and revision of the manuscript; Andrés Flores García: drafting and editing of the manuscript; Vidal Guerra de la Cruz: data collection, drafting and editing of the manuscript.

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