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Research Article

Sistemas compatibles de ahusamiento y volumen comercial para dos especies de *Pinus* en Durango, México

Taper and merchantable volume systems for two *Pinus* species in *Durango*, Mexico

Francisco Cruz-Cobos[1], Gerónimo Quiñonez-Barraza[2]*, Verónica Hernández-Merino[3], Sacramento Corral-Rivas[1], Adan Nava-Nava[4]

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[1]Instituto Tecnológico de El Salto, Durango. México.
[2]INIFAP, Centro de Investigación Regional Norte Centro, Campo Experimental Valle del Guadiana. México.
[3]Universidad Autónoma de Nuevo León, Facultad de Ciencias Forestales. México.
[4]Colegio de Postgraduados, Campus Montecillo, Postgrado en Ciencias Forestales. México.

*Autor para correspondencia; correo-e: quinonez.geronimo@inifap.gob.mx
*Corresponding author; e-mail: quinonez.geronimo@inifap.gob.mx

Resumen

El estudio tuvo como objetivo comparar y validar dos metodologías para estimar el volumen comercial de *Pinus cooperi* (*Pc*) y *Pinus durangensis* (*Pd*) en la región forestal de El Salto, Durango, México. Los datos de ahusamiento y volumen utilizados fueron de 164 y 182 árboles de *Pc* y *Pd*, respectivamente; a los cuales se les midió el diámetro a diferentes alturas. De la base de datos, 70 % se usó para el ajuste y 30 % para la validación. Cuatro sistemas compatibles de ahusamiento y volumen comercial fueron ajustados: tres de razón de volúmenes y uno segmentado. El ajuste se realizó mediante regresión iterativa aparentemente no relacionada. Los criterios para evaluar la calidad de ajuste fueron: el coeficiente de determinación ajustado, la raíz del cuadrado medio del error, el sesgo, el criterio de información de *Akaike* y el coeficiente de variación. Los estadísticos de ajuste y de validación indicaron que el sistema segmentado (SS1) fue el más preciso para estimar el volumen comercial y ahusamiento para ambas especies. El sistema basado en modelos de razón de volúmenes (RVS2) generó resultados similares, constituyó un sistema con más parsimonia, y explicó 98.3 % de la variabilidad observada en *Pc* y 97.6 % en *Pd*, y podría ser más fácil de implementar que el segmentado para predecir la altura del fuste, el diámetro del fuste, el volumen comercial y el volumen total del árbol para los dos taxones estudiados. Por tanto, se recomienda el sistema etiquetado como RVS2 por su precisión y simplicidad para estimar volúmenes comerciales de las especies evaluadas.

Palabras claves: Ahusamiento, modelos de razón de volúmenes, parsimonia, sistema compatible, volumen comercial, volumen total.

Abstract

The objective of the study was to compare and validate two methodologies in order to estimate the merchantable volume of *Pinus cooperi* (*Pc*) and *Pinus durangensis* (*Pd*) in the forest region of *El Salto*, *Durango*, Mexico. The used data were 164 *Pc* and 182 *Pd* trees, whose diameters at different heights and the total height were measured. The 70 % of the dataset was used for training, and 30 % was utilized for testing. Four compatible taper and merchantable volume systems were fitted, three of which were based on volume ratio models, and one, on a segmented taper model. The fitting was performed through iterative seemingly unrelated regression. The criteria for evaluating predictive performance were the adjusted coefficient of determination, the root mean square error, the bias, the Akaike's information criterion, and the coefficient of variation. The training and testing statistics indicated that the segmented system (SS1) was the most accurate for estimating the merchantable volume and taper of both species. The system based on volume ratio models (RVS2) generated similar results, was more parsimonious and explained 98.3 % of the observed variability in *Pc* and 97.6 % of that observed in *Pd*; furthermore, it may prove easier to implement than segmented system for predicting the stem height, stem diameter, merchantable volume, and total tree volume for two studied species. The system labeled RVS2 is recommended for its accuracy and simplicity in estimating the merchantable volume of the evaluated species.

Key words: Tapering, volume ratio models, parsimony, compatible system, merchantable volume, total volume.

Resumen

El estudio tuvo como objetivo comparar y validar dos metodologías para estimar el volumen comercial de *Pinus cooperi* (*Pc*) y *Pinus durangensis* (*Pd*) en la región forestal de El Salto, Durango, México. Los datos de ahusamiento y volumen utilizados fueron de 164 y 182 árboles de *Pc* y *Pd*, respectivamente; a los cuales se les midió el diámetro a diferentes alturas. De la base de datos, 70 % se usó para el ajuste y 30 % para la validación. Cuatro sistemas compatibles de ahusamiento y volumen comercial fueron ajustados: tres de razón de volúmenes y uno segmentado. El ajuste se realizó mediante regresión iterativa aparentemente no relacionada. Los criterios para evaluar la calidad de ajuste fueron: el coeficiente de determinación ajustado, la raíz del cuadrado medio del error, el sesgo, el criterio de información de *Akaike* y el coeficiente de variación. Los estadísticos de ajuste y de validación indicaron que el sistema segmentado (SS1) fue el más preciso para estimar el volumen comercial y ahusamiento para ambas especies. El sistema basado en modelos de razón de volúmenes (RVS2) generó resultados similares, constituyó un sistema con más parsimonia, y explicó 98.3 % de la variabilidad observada en *Pc* y 97.6 % en *Pd*, y podría ser más fácil de implementar que el segmentado para predecir la altura del fuste, el diámetro del fuste, el volumen comercial y el volumen total del árbol para los dos taxones estudiados. Por tanto, se recomienda el sistema etiquetado como RVS2 por su precisión y simplicidad para estimar volúmenes comerciales de las especies evaluadas.

Palabras claves: Ahusamiento, modelos de razón de volúmenes, parsimonia, sistema compatible, volumen comercial, volumen total.

Introduction

The volume of trees both individually and at stand level, is a very important component in the management of forest resources, as it allows planning, executing and evaluating activities proposed in timber forest management plans (Alder, 1980; Prodan *et al*., 1997; Cruz-Cobos *et al.*, 2008).

Volumetric forest stocks represent a reliable criterion for measuring the productive potential of a stand (Burkhart and Tomé, 2012; Avery and Burkhart, 2015) and can be divided into total tree volume (*Vt*, m^3) and merchantable volume (*Vm*, m^3)

(Chauchard and Sbrancia, 2005); the latter makes it possible to carry out a distribution of products and thus determine the economic value of the stands.

In forest modeling, there are three methodologies for estimating the *Vm* (Prodan *et al.,* 1997): (1) *Vt* functions with the constraint of a limiting diameter or height, (2) functions describing the tree profile and estimations of the *Vm* at a defined diameter or a given height (Chauchard and Sbrancia, 2005), and (3) ratio volume functions (*Rv*) (Burkhart, 1977; Burkhart and Tomé, 2012) corresponding to the ratio between the *Vm* and the *Vt* according to a given height limit (Barrios *et al*., 2014; Tamarit *et al*., 2014; Quiñonez-Barraza *et al.*, 2019).

Volume ratio functions have been successfully used in conifers and broadleaves because of the accuracy and simplicity with which they are generated (Barrios *et al*., 2014; Hernández-Ramos *et al.*, 2017, 2018; Niño *et al.*, 2018). One of the advantages of this type of function is that it minimizes volume estimation errors directly (Prodan *et al*., 1997; Quiñonez-Barraza *et al.*, 2019).

Lynch *et al.* (2017) and Zhao and Kane (2017) proposed a series of equations to estimate the *Vm* based on the ratio of volume to height, which have been fitted with satisfactory results for several tree species (García-Espinoza *et al.*, 2018; Quiñonez-Barraza *et al.*, 2019; Castillo-López *et al.*, 2021). Therefore, it is easier to derive taper equations from the volume ratio and total height (*H*, m), in contrast to the volume and diameter ratio (Quiñonez-Barraza *et al.*, 2019; Zhao *et al*., 2019). The resulting volume and taper equations are mutually compatible (Zhao *et al.*, 2019; Castillo-López *et al.*, 2021).

Bi and Long (2001) and Özçelik and Göçeri (2015) point out that volume and taper equations must be generated because of site quality, stand density, and silvicultural treatments have a direct effect on tree profile (Burkhart and Tomé, 2012; Quiñonez-Barraza *et al*., 2014; Özçelik and Cao, 2017; Castillo-López *et al.*, 2021). In this regard, two species (*Pinus cooperi* C. E. Blanco and *Pinus durangensis* Martínez) which, due to their distribution and abundance in the region of *El Salto*, *Durango*, contribute with more than 80 % of the timber forest production were selected for a case study (González-Elizondo *et al.*, 2012).

The objectives were to fit and validate three volume-taper systems based on volume ratio equations and to compare the performance with a segmented merchantable taper-volume system for *Pinus cooperi* and *Pinus durangensis* in *Pueblo Nuevo*, *Durango*, Mexico.

Materials and Methods

Study area

The study was carried out in natural stands in the *Banderas del Águila*, *Chavarría Nuevo*, *Chavarría Viejo*, *El Brillante*, *El Salto*, *La Campana*, *La Ciudad*, *La Victoria*, *Mil Diez*, and *San Esteban ejidos* in the *Pueblo Nuevo* municipality, state of *Durango*, Mexico (Figure 1), all of which are located within Unit of Forest Management (Umafor, by its acronym in Spanish) 1 008 "*El Salto*" in the southwest of *Durango* state. The altitude fluctuates between 1 400 and 2 600 m. The vegetation conditions are those of pure oak and pine stands, but mostly of mixed pine-oak forests. According to Köppen's classification as modified by García (2004), the region is within the temperate climate group C, the subgroup of semi-warm climates (A)C(w_1) and semi-warm sub-humid types, with summer rainfall and a percentage of winter rainfall between 5 and 10.2 %, and with an average annual rainfall of 800 to 1 200 mm and an average annual temperature of 20 to 22 °C.

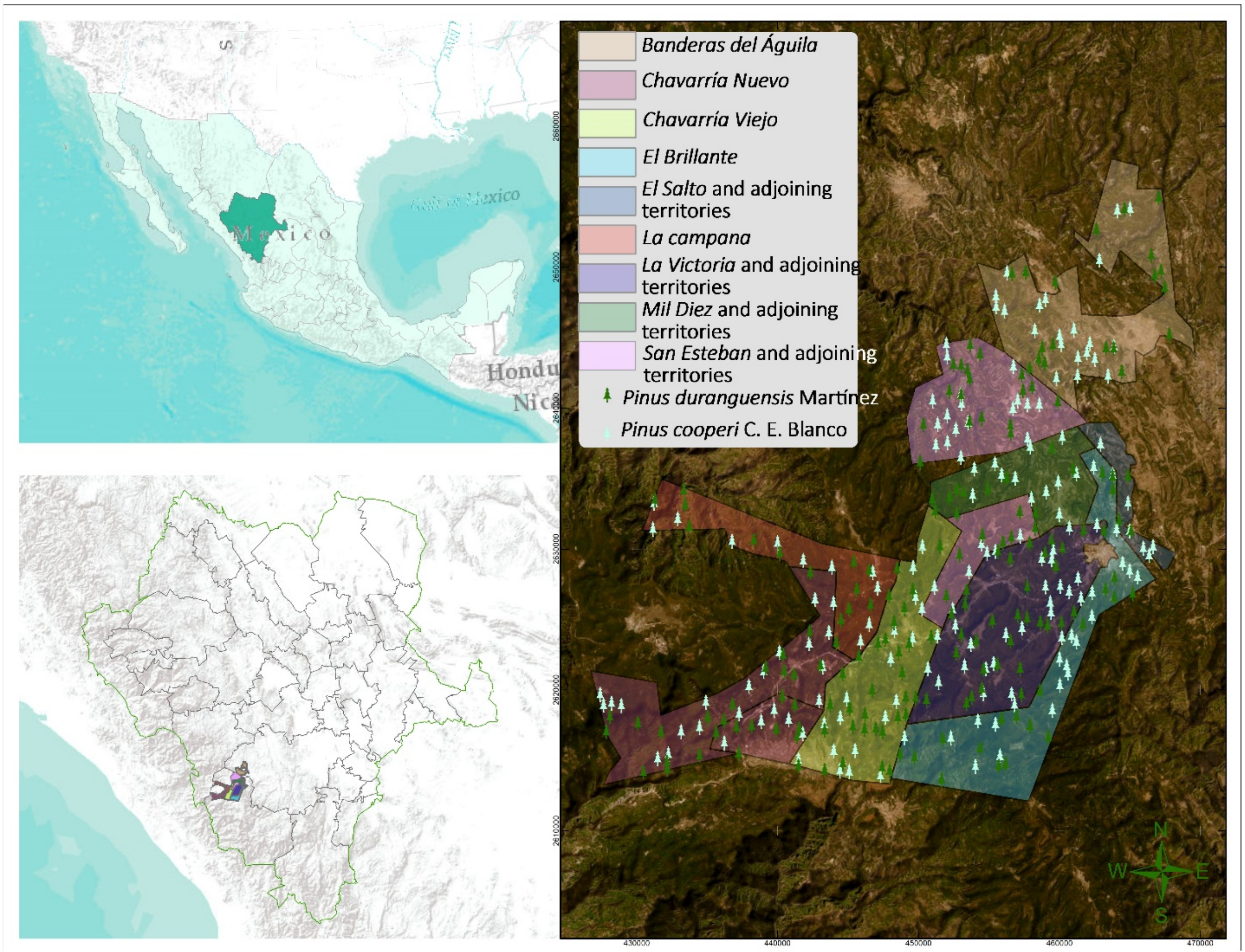

Figure 1. Geographical location of the study area.

Database

Based on a destructive sampling and taper analysis during the timber harvesting of the selected forests communities, 164 *Pinus cooperi* trees and 182 *Pinus durangensis* trees were felled and measured. The measured variables were outside-bark diameter at breast height (*D*, cm) (with model 283D/5M Forestry Suppliers® diameter tape), total height (*H*, m) with model Nikon Forestry PRO II Nikon® hypsometer, outside-bark diameter (*d*, cm) (with the same diameter tape) at different heights (*h*), and merchantable height (*h*, m). The first selection was at stump height (*hs*, m); subsequently, three 0.30 m sections were obtained (to capture the taper below 1.3 m of height); the next section corresponded to the *D*, followed by sections of 2.54 m in length (measuremed with model TP30ME Truper® long tape) until reaching the tip of the tree (*Th*, m). The volume of each section was calculated with the neiloid formula for the stump ($Vs = \frac{S_0 l}{4}$), the Smalian formula ($Vs = \frac{S_0 + S_1}{2} l$) for the intermediate or neiloid sections and cone shapes ($Vs = \frac{S_0}{3} l$) for tip of the tree (Barrios *et al.*, 2014; Avery and Burkhart, 2015). In all cases, Vs is the volume of the section, S_0 is the area of the initial cross-section, S_1 is the area of the final cross-section, and l is the length of the log. The total volume of the stem was obtained by adding the volumes of the sections determined for each tree. The dataset was divides in 70 % for fitting, and 30 % for testing.

Assessed models

Three systems of volume ratio equations were fitted they were developed by Lynch *et al.* (2017) and based on the volume ratio equations cited by Zhao and Kane (2017) (RVS1-RVS3), in addition to the Segmented Tapering-Volume System (SS1) developed by Fang *et al.* (2000) (Table 1). The comparison was made based on the premise that a system of equations based on volume ratio expressions is simpler and more practical than a segmented model.

Table 1. Assessed compatible systems of volume-taper equations.

System	Equation
RVS1	$Vm = \alpha_0 D^{\alpha_1} H^{\alpha_2}\left[1-(1-\frac{h}{H})^{\beta_1}\right]$
	$d = \sqrt{\frac{\beta_1 \alpha_0 D^{\alpha_1} H^{\alpha_2}}{Hk}\left(1-\frac{h}{H}\right)^{\beta_1-1}}$
RVS2	$Vm = \alpha_0 D^{\alpha_1} H^{\alpha_2}\left[1-(1-\frac{h}{H})^{\beta_1}\right]^{\beta_2}$
	$d = \sqrt{\frac{\alpha_0 D^{\alpha_1} H^{\alpha_2}\beta_1\beta_2}{Hk}\left(1-\left(1-\frac{h}{H}\right)^{\beta_1}\right)^{\beta_2-1}\left(1-\frac{h}{H}\right)^{\beta_1-1}}$
RVS3	$Vm = \alpha_0 D^{\alpha 1} H^{\alpha 2}\left[1-\left(1-\frac{h}{H}\right)^{\beta_1}\right]^{\{1-\beta_2 exp[-\exp(\beta_3 D^{\alpha 1} H^{\alpha 2})]\}}$
	$d = \sqrt{\frac{\beta_1}{kH}\alpha_0 D^{\alpha_1} H^{\alpha_2}\left(1-\frac{h}{H}\right)^{\beta_1-1}\{1-\beta_2 exp[-exp(\beta_3 D^{\alpha_1} H^{\alpha_2})]\}\left[1-\left(1-\frac{h}{H}\right)^{\beta_1}\right]^{\{-\beta_2 exp[-\exp(\beta_3 D^{\alpha 1} H^{\alpha 2})]\}}}$
SS1	$d = c_1\sqrt{H^{\frac{k-\beta_1}{\beta_1}}\left(1-\frac{h}{H}\right)^{\frac{k-\beta}{\beta}} A_1^{I_1+I_2} A_2^{I_2}}$
	$Vm = c_1^2 H^{\frac{k}{\beta_1}}\left[\beta_1 r_o + (I_1 + I_2) + (\beta_2-\beta_1) r_1 + I_2(\beta_3-\beta_2) A_1 r_2 - \beta(1-h/H)^{\frac{k}{\beta}} A_1^{I_1+I_2} A_2^{I_2}\right]$
	$c_1 = \sqrt{\frac{\alpha_0 D^{\alpha_1} H^{\alpha_2-\frac{k}{\beta_1}}}{\beta_1(r_o-r_1)+\beta_2(r_1-A_1 r_2)+\beta_3 A_1 r_2}}$
	$r_0 = \left(1-\frac{hs}{H}\right)^{\frac{k}{\beta_1}};\ r_1 = (1-p_1)^{\frac{k}{\beta_1}}; r_2 = (1-p_2)^{\frac{k}{\beta_2}}$

$$\beta = \beta_1^{1-(I_1+I_2)}\beta_2^{I_1}\beta_3^{I_2}; A_1 = (1-p_1)^{\frac{(\beta_2-\beta_1)k}{\beta_1\beta_2}}; A_2 = (1-p_2)^{\frac{(\beta_3-\beta_2)k}{\beta_2\beta_3}}$$

$$\text{Where } \begin{cases} I_1 = 1 \; if \; p_1 \le \frac{h}{H} \le p_2; \text{ otherwise}, 0 \\ I_2 = 1 \; if \; p_2 \le \frac{h}{H} \le 1; \text{ otherwise}, 0 \end{cases}$$

$$p_1 = \frac{h_1}{H}; \; p_2 = \frac{h_2}{H}$$

D = outside-bark diameter at breast height (cm); *d* = Diameter (cm) at height *h* (m); *h* = Merchantable height (*Mh*, m); *H* = Total height (m); *Sh* = Stump height (m); *Vm* = Merchantable volume (m^3); $k = \pi/40\,000$; α_i and β_i = Parameters to be estimated; p_1 = Inflexion point for the change from neiloid to paraboloid; p_2 = Infexion point for the change from paraboloid to cone; I_1 = Indicator variable for infexion point p_1; I_2 = Indicator variable for inflexion point p_2.

Fitting of systems and selection criteria

Parameter estimation was performed with iterative seemingly unrelated regression (*ITSUR*), using the MODEL procedure of the SAS/ETS® 9.4 statistical package (Statistical Analysis System, 2019). Fang *et al.* (2000) indicate that the fitting with *ITSUR* homogenizes and minimizes the standard error of the parameters in the system and allows for system compatibility. In order to avoid convergence in parameter estimation —especially if *h*=*H*, i.e., when *d*=0—, a value of *h*=0.001 was applied, together with an dummy variable at the tip to prevent estimating the partial derivatives of the parameters at zero.

The quantitative evaluation of the fitted systems, both in the training (70 % of the database) and in the testing (30 % of the database), was based on the adjusted coefficient of determination (R^2a, 1), root mean square error (*RMSE*, 2), coefficient of variation (*CV*, 3), average bias ($\bar{E}$, 4), and Akaike information criterion (*AIC*, 5). These statistics have been used in the assessment of taper and merchantable volume models (Quiñonez-Barraza *et al.*, 2019; Zhao *et al.*, 2019).

$$R^2a = 1 - \left[\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2/(n-p)}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2/(n-1)}\right] \quad (1)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-1}} \quad (2)$$

$$CV = \sqrt{\frac{\sum_{i=1}^{n}\frac{(y_i - \hat{y}_i)^2}{n-1}}{\bar{y}}} \quad (3)$$

$$\bar{E} = \sum_{i=1}^{n}(y_i - \hat{y}_i)/n \quad (4)$$

$$AIC = -2\log(L) + 2p \quad (5)$$

Where:

y_i = Observed value of *d* or *Vm*

$\hat{y}_i$ = Estimated value of *d* or *Vm*

$\bar{y}$ = Mean observed value of *d* or *Vm*

n = Number of observations

p = Number of system parameters

$log(L)$ = Logarithm of likelihood

In order to assess the accuracy of the equations, a ranking criterion was generated based on Sakici *et al.* (2008), where the number 1 is assigned to the equation with the best statistic of fitting and 4 to the worst statistic of fitting. The sum of the values formed the total ranking, i.e., the equation with the lowest value in the total ranking was identified as the most accurate. This procedure was also used in the validation of the taper and merchantable volume equations of the generated systems. Once the systems were fitted, predictions were made for the validation database and testing or validation statistics were obtained.

The autocorrelation in the taper component was corrected with a continuous second-order autocorrelation error structure (*CAR_2*) that considered the distance between measurements of the merchantable height in each tree (Zimmerman and Núñez-Antón, 2001), as these structures have been proven to be functional in taper and merchantable volume equations (Corral-Rivas *et al.*, 2017; García-Espinoza *et al.*, 2018; Castillo-López *et al.*, 2021). The autocorrelation structure was not included in the merchantable volume equation, as it does not represent a gain in the fitted equation (Quiñonez-Barraza *et al.*, 2019; Zhao *et al.*, 2019).

$$e_{ij} = d_1 {\gamma_1}^{h_{ij}-h_{ij-1}} e_{ij-1} + d_2 {\gamma_2}^{h_{ij}-h_{ij-2}} e_{ij-2} + \varepsilon_{ij} \qquad (6)$$

Where:

e_{ij} = Ordinary residual in tree *i*

d_i = 1 for *j*>1

d_i = 0 for *j*=1

γ_1 = autocorrelation parameter of order *i*

$h_{ij} - h_{ij-1}$ = Distance from *j* to *j-1* observation inside each tree $h_{ij} > h_{ij-1}$

$h_{ij} - h_{ij-2}$ = Separation distance from *j* to *j-2* observation inside each tree $h_{ij} > h_{ij-2}$

The correction of the heteroscedasticity associated with the merchantable volume was carried out by means of a function that weighted the variance of the residuals with the specification utilized by Quiñonez-Barraza *et al.* (2014):

$$resid.Vm = resid.\frac{Vm}{[(D^2H)^{\omega}]^{0.5}}$$

In which the value of the parameter ω is derived from the linear regression of the natural logarithm of the residuals based on the combined variable (D^2H).

Results

Fitting stage

Table 2 shows the fitting of the volume and taper equations for the studied species. The SS1 was observed to provide the best fitting to the stem profile with the lowest score in both species, and explained more than 98.2 % of the observed variability. SS1 was the most accurate in *Pinus cooperi*, determining *RMSE* values for volume and taper lower by 19 and 9 %, respectively, than those obtained for *Pinus durangensis* (Table 2). The second place in accuracy was for RVS2, which accounted for more than 97.5 % of the variability observed for both species (Table 2); these equations are simpler than the segmented taper-volume system of Fang *et al.* (2000).

Table 2. Adjustment statistics of the evaluated systems of merchantable volume-drawdown (*Vm-d*) for *Pinus cooperi* C. E. Blanco (*Pc*) and *Pinus durangensis* Martínez (*Pd*).

Species	System	Component	*RMSE*	R^2a	*AIC*	*CV*	$\bar{E}$	*R*	$\Sigma Mc+d$
Pc	RVS1	*Vm*	0.101	0.98	-6422.6	15.7	-0.045	17	36
		d	2.826	0.97	2927.1	10.6	0.311	19	
	RVS2	*Vm*	0.074	0.99	-7290.1	12.4	-0.021	10	22
		d	2.145	0.98	2152.0	8.0	0.109	12	
	RVS3	*Vm*	0.075	0.99	-7275.0	12.5	-0.021	15	24
		d	2.138	0.98	2142.7	8.0	0.202	9	
	SS1	*Vm*	0.063	0.99	-11115.3	10.3	0.019	8	18
		d	2.035	0.99	2865.8	7.3	0.583	10	
Pd	RVS1	*Vm*	0.110	0.98	-7176.2	16.0	-0.046	16	36
		d	3.656	0.95	4231.4	13.4	0.763	20	
	RVS2	*Vm*	0.086	0.99	-7971.7	13.3	-0.025	13	23
		d	2.608	0.98	3134.2	9.7	0.275	10	
	RVS3	*Vm*	0.835	0.99	-8062.5	13.2	-0.017	13	28
		d	2.682	0.97	3226.8	9.8	0.547	15	
	SS1	*Vm*	0.078	0.99	-8281.0	12.5	0.005	8	13
		d	2.245	0.98	2652.8	8.4	0.048	5	

RMSE = Root mean square error; R^2a = Adjusted coefficient of determination; *AIC* = Akaike information criterion; *CV* = Coefficient of variation; $\bar{E}$ = Bias; *R* = Ranking of the system; *Vm* = Merchantable volumen; *d* = diameter or taper.

The RVS2 registered goodness-of-fit statistics similar to those of SS1 for *Pinus cooperi*; in this case, the estimated error in volume increased by a comparative 18 %, while the error in describing the forest profile increased by only 5 %. The accuracy of RVS2 in *Pinus durangensis* and *Pinus cooperi* was similar, but the *RMSE* value for estimating volume and describing taper was increased by 10 % and 16 %, respectively. In contrast, RVS1 had the worst performance with the highest values of the rating established in the goodness-of-fit statistics for estimating the merchantable volume and describing the forest profile of both species.

Table 3 shows the estimated parameter values for the two species studied; all parameters were different from zero at a significance level of 5 %. For *Pinus cooperi*, it was observed that the tipping points for the segmented system SS1 occur at 4.31 % of the tree height, near the base, and at 69.87 % of the total height. While in *Pinus durangensis*, the tipping points are recorded at 4.36 % and 74.87 % of the relative height of the stem.

Table 2. Parameter estimators obtained with the simultaneous fitting of taper and merchantable volume for the two *Pinus* species studied.

Species	**System**	**Parameters**	**Estimation**	**Standard error**	***T* value**	***Pr*>|*t*|**
Pc	RVS1	α_0	0.000067	2.67×10^{-6}	24.99	<0.0001
		α_1	1.922239	1.05×10^{-2}	182.59	<0.0001
		α_2	0.955169	1.04×10^{-2}	91.77	<0.0001
		β_1	2.460480	2.11×10^{-2}	116.81	<0.0001

	RVS2	α_0	0.000066	2.47×10^{-6}	26.65	<0.0001
		α_1	1.916322	9.88×10^{-3}	193.93	<0.0001
		α_2	0.965511	1.01×10^{-2}	95.83	<0.0001
		β_1	2.071045	1.68×10^{-2}	123.11	<0.0001
		β_2	0.956080	1.33×10^{-3}	717.60	<0.0001
	RVS3	α_0	0.000061	2.39×10^{-6}	25.71	<0.0001
		α_1	1.929970	1.01×10^{-2}	191.34	<0.0001
		α_2	0.971030	1.01×10^{-2}	96.60	<0.0001
		β_1	2.112498	1.64×10^{-2}	128.92	<0.0001
		β_2	0.108429	3.96×10^{-3}	27.36	<0.0001
		β_3	0.000010	8.88×10^{-7}	11.27	<0.0001
	SS1	α_0	0.000063	1.71×10^{-6}	36.50	<0.0001
		α_1	1.948812	7.24×10^{-3}	269.17	<0.0001
		α_2	0.938103	9.67×10^{-3}	97.05	<0.0001
		β_1	0.000006	1.65×10^{-7}	35.93	<0.0001
		β_2	0.000043	3.78×10^{-7}	112.56	<0.0001
		β_3	0.000030	4.14×10^{-7}	71.28	<0.0001
		p_1	0.043174	1.15×10^{-3}	37.40	<0.0001
		p_2	0.698774	9.20×10^{-3}	75.96	<0.0001
Pd	RVS1	α_0	0.000085	3.39×10^{-6}	22.62	<0.0001
		α_1	1.887200	1.03×10^{-2}	164.53	<0.0001
		α_2	0.907800	9.89×10^{-3}	82.70	<0.0001
		β_1	2.477400	2.12×10^{-2}	105.26	<0.0001
	RVS2	α_0	0.000084	3.14×10^{-6}	24.08	<0.0001
		α_1	1.881500	9.70×10^{-3}	174.75	<0.0001
		α_2	0.919500	9.60×10^{-3}	86.35	<0.0001
		β_1	2.070300	1.68×10^{-2}	110.94	<0.0001

	β_2	0.952100	1.33×10^{-3}	646.63	<0.0001
RVS3	α_0	0.000076	2.98×10^{-6}	23.00	<0.0001
	α_1	1.900100	9.93×10^{-3}	172.42	<0.0001
	α_2	0.925100	9.58×10^{-3}	87.04	<0.0001
	β_1	2.114600	1.64×10^{-2}	116.17	<0.0001
	β_2	0.128800	4.71×10^{-3}	24.65	<0.0001
	β_3	0.000013	1.15×10^{-6}	10.15	<0.0001
SS1	α_0	0.000073	1.99×10^{-6}	33.12	<0.0001
	α_1	1.917700	7.12×10^{-3}	242.55	<0.0001
	α_2	0.916200	9.44×10^{-3}	87.45	<0.0001
	β_1	0.000005	1.38×10^{-7}	32.73	<0.0001
	β_2	0.000042	3.70×10^{-7}	102.38	<0.0001
	β_3	0.000030	4.14×10^{-7}	65.28	<0.0001
	p_1	0.043600	1.17×10^{-3}	33.70	<0.0001
	p_2	0.748700	9.86×10^{-3}	68.45	<0.0001

Pc = *Pinus cooperi* C. E. Blanco; *Pd* = *Pinus durangensis* Martínez.

Validation

Table 4 shows the testing or validation statistics. RVS2 obtained the best score for *Pinus cooperi*, both RVS2 and SS1 had similar scores. Accuracy in terms of the *RMSE* value in the prediction of the volume of the forest was 6% lower than that estimated with the SS1, while it was 13 % higher for describing the taper. Looking at the graphical behavior of the mean bias, the RVS2 and RVS4 underestimate both variables of interest (figures 2a and 2c, figures 3a and 3c).

Table 4. Validation statistics of the merchantable volume-taper (*Vm-d*) equations for *Pinus cooperi* C. E. Blanco (*Pc*) and *Pinus durangensis* Martínez (*Pd*).

Species	System	Component	*RMSE*	R^2a	*AIC*	*CV*	$\bar{E}$	*R*	$\Sigma Vc+d$
Pc	RVS1	*Vm*	0.011	0.98	-2719.1	17.7	-0.023	14	34
		d	15.628	0.94	1675.0	14.2	1.044	20	
	RVS2	*Vm*	0.009	0.99	-2717.1	15.9	0.001	9	19
		d	7.738	0.97	1251.0	10.1	0.555	10.5	
	RVS3	*Vm*	0.009	0.99	-2715.1	16.0	-0.001	12	26
		d	8.222	0.97	1290.0	10.1	0.669	14.5	
	SS1	*Vm*	0.009	0.98	-2711.1	15.8	0.026	15	20
		d	6.746	0.98	1174.0	9.5	0.496	5	
Pd	RVS1	*Vm*	0.112	0.98	-3048.8	15.6	-0.065	17	37
		d	3.554	0.95	1783.0	13.3	0.365	20	
	RVS2	*Vm*	0.083	0.99	-3467.7	12.0	-0.043	14	24
		d	2.514	0.98	1301.7	9.5	-0.114	10	
	RVS3	*Vm*	0.077	0.99	-3565.5	11.6	-0.035	11	26
		d	2.716	0.97	1411.7	10.2	0.283	15	
	SS1	*Vm*	0.065	0.99	-3805.0	10.8	-0.014	8	13
		d	2.231	0.98	1141.5	8.3	-0.390	5	

RMSE = Root mean square error; R^2a = Adjusted coefficient of determination; *CV* = Coefficient of variation; $\bar{E}$ = Average error; *AIC* = Akaike information criterion; *R* = Ranking of the model.

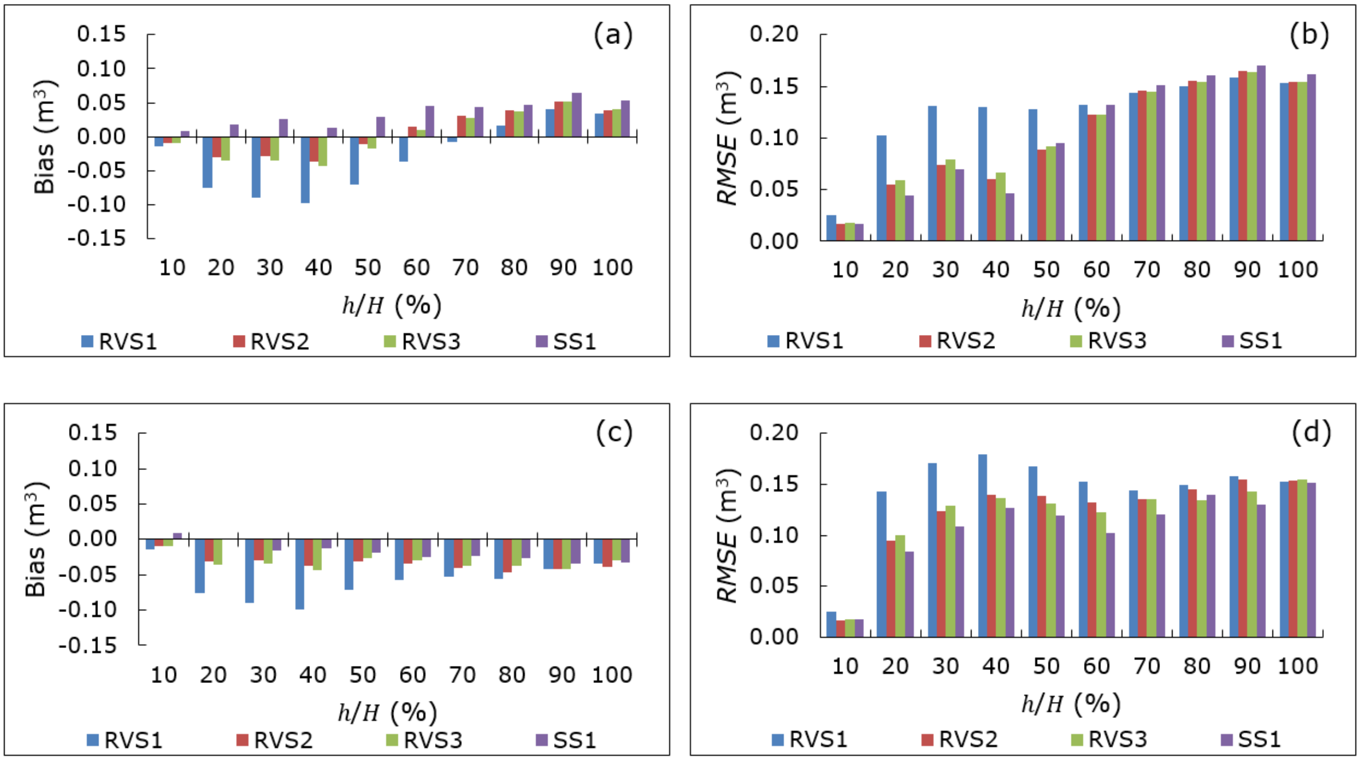

Figure 2. Evolution of the average error and RMSE by relative height (h/H) in the estimation of the stem volume in Pinus cooperi C. E. Blanco (a) and (b), and Pinus durangensis Martínez (c) and (d).

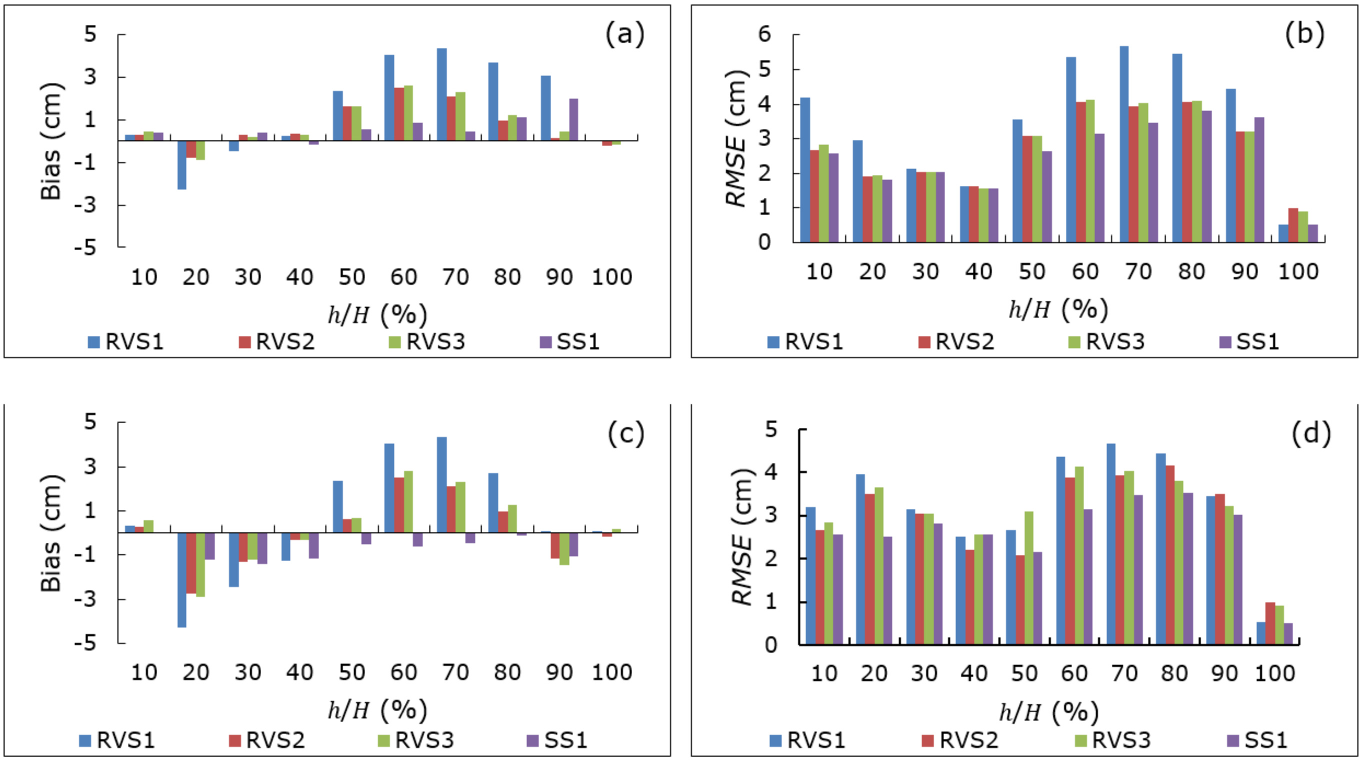

Figure 3. Evolution of average error and RMSE by relative height (h/H) for taper in Pinus cooperi C. E. Blanco (a) and (b), and in Pinus durangensis Martínez (c) and (d).

In the validation process for *Pinus durangensis*, SS1 generated the best predictive performance, followed by RVS2 which exhibited a 22 % higher estimated error (*RMSE*) in volume, while its *RMSE* for taper was 11 %. Both systems explain more than 97.6 % of the observed variability, and, based on the average error, overestimate both volume and taper (Table 4). In contrast, the lowest predictive performance was recorded with RVS1.

Figure 2a shows the behavior of the error in volume with respect to the relative height in the fitting of the four systems of equations. In the *Pinus cooperi* species, the average error committed with the RVS1-RVS3 systems had the same behavior, i.e., they overestimated the volume of the forest, in contrast to SS1, which underestimated the merchantable volume (Figure 2a). The evolution of the *RMSE* value along the relative stem height confirmed that both the RVS2 and SS1 systems satisfactorily predicted the merchantable volume, with values below 0.2 m^3 (Figure 2b). In *Pinus durangensis*, the RVS1-RVS3 systems also overestimated the volume of the stem based on its height, while the SS1 system exhibited this behavior only after 30 % of the stem height (Figure 2c). The behavior of the *RMSE* value confirmed, once again, that RVS2 and SS1 satisfactorily predicted the volume of the stem with errors smaller than 0.15 m^3 (Figure 2d).

Figure 3 shows the evolution of the mean bias value and *RMSE* by relative height (h/H) to describe the taper of *Pinus cooperi* (Figure 3a and 3b). The RVS1 and RVS3 systems underestimated the diameter in relation to the stem height with one exception at 30 % of the stem height. RVS1 registered values of up to 5 cm at relative heights above 80 %, and the lowest values were less than 2 cm (Figure 3a). The *RMSE* values in the RVS2-RVS3 systems confirmed a higher accuracy at stem heights, being lower at a stem height of 4 cm, while a low accuracy was obtained at stem heights above 40 % of the total height (Figure 3b).

In *Pinus durangensis* (Figure 3c and 3d), based on the evolution of the average bias with respect to the relative height, systems RVS1, RVS2, and RVS3 were determined to underestimate the diameter at 10 to 20 % and at 60 to 100 % of the relative height; in contrast, they overestimated at 20 to 60 %. On the other hand, SS1 tends to slightly overestimate the diameters at 30 % of the relative height (Figure 3c). The *RMSE* value for RVS2-RVS4 along the stem was <3 cm, while RVS1 can reach values close to 5 cm (Figure 3d).

Discussion

The results indicated that the system of Fang *et al.* (2000) (SS1) was more accurate for estimating stem volume and describing taper in both species (Table 2). This is consistent with the systems developed by Silva-González *et al.* (2018) in *Chihuahua*, by Cruz-Cobos *et al.* (2008) in *Durango*, and by Quiñonez-Barraza *et al.* (2014) for *Pinus durangensis*, *Pinus cooperi*, and *Pinus arizonica* Engelm., mainly these authors conclude that the equation of Fang *et al.* (2000) is also more accurate in regard to other assessed functions.

The equation of Fang *et al.* (2000) provides a very likely description of the stem profile, and its incorporation allows estimating the volume of the stem at any given height or diameter; therefore, it is an important tool for the elaboration and execution of forest management programs (Corral-Rivas *et al.*, 2007).

Based on the particular analysis of SS1, the inflexion points for both species exhibited similarities; for example, the first tipping point for *Pinus cooperi* occurs at 4.31 % of the tree height, near the base, and is very similar to that estimated for *Pinus durangensis* (at 4.36 % of the tree height). However, there are notable differences in the height of the tree where the second inflexion point occurs, which is 69.87 % for *Pinus cooperi* and 74.87 % for *Pinus durangensis*; this represents a gain of 5 % in product obtained from the stem with desirable characteristics for the industry (Table 2).

The percentage values referred to above are lower than those recorded by Corral-Rivas *et al.* (2007) in the region of *El Salto*, *Durango*, for *Pinus cooperi* (6.64 and 73.80 %), but they are consistent with those registered by Corral-Rivas *et al.* (2017) at state level (4.61 and 71.33 %). These differences can be ascribed to the databases used for the construction of the taper and merchantable volume equations. Similarly, Silva-González *et al.* (2018) obtained similar results to those cited for *Pinus durangensis* in Forest Management Unit 0808 in the state of *Chihuahua*, Mexico (4.84 and 75.21 %). However, these values are higher than those reported by the following authors:

Corral-Rivas *et al.* (2007) for the *El Salto* region of *Durango*, with inflexion points at 5.6 and 69.2 %, Corral-Rivas et *al.* (2017) at 2.3 and 73.53 %, and Quiñonez-Barraza *et al.* (2014) at 4.7 and 70.9 % for the *Durango* state.

The RVS2 ratio equation was determined to be more accurate for the two species studied, with errors of less than 0.09 m^3 for the estimation of the stem volume and less than 2.6 cm for the diameter at any height of the stem (Table 2). In comparison, a higher accuracy was observed, but only marginally when estimating the volume of the forest with respect to SS1, which has a more complex mathematical structure. The ratio models recorded by Lynch *et al.* (2017), like those of Zhao and Kane (2017) may be regarded as simpler expressions for estimating the volume and diameter of trees; in addition, the error estimated in this work was similar to that calculated by the equation of Fang *et al.* (2000).

Volume ratio equations have important advantages over compatible equations for the calculation of total volume, merchantable volume, and stem diameters in various *Pinus* species. García-Espinoza *et al.* (2018) developed a compatible system based on volume ratio equations to estimate diameter, stem, total, and branch volume for *Pinus pseudostrobus* Lindl. in commercial forest plantations in *Michoacán*, Mexico.

Based on the results obtained in the validation stage and considering certain works with different species of conifers and broadleaf trees (Prodan *et al.*, 1997; Chauchard and Sbrancia, 2005; Fonweban *et al.*, 2012; Barrios *et al.*, 2014; Hernández-Ramos *et al.*, 2017, 2018; Zhao *et al.*, 2019), as well as on the validation statistics, the RVS2 system is recommended as a simple and reliable alternative for estimating total stem and merchantable volume, stem heights and diameters for *Pinus cooperi* and *Pinus durangensis* in the *Pueblo Nuevo* region of *Durang*o, Mexico, as a tool for quantifying their volumes.

The RVS2 system generated consistent results and showed accurate estimates of stem volume and diameter (Tables 2 and 4), without implementing numerical integration methods (Chauchard and Sbrancia, 2005). Therefore, although the models based on volume ratio equations are inferior in the fit statistics for the taper equations, they show goodness of fit in the merchantable volume equations for both *Pinus cooperi*, with very similar values for *Pinus durangensis*.

Conclusions

In general terms, the volume estimates and stem diameters provided for *Pinus cooperi* and *Pinus durangensis* by the merchantable volume-taper system labeled RVS2, which is based on a volume ratio equation, are similar to those obtained with the SS1 system, which is based on a taper model. However, due to its lower complexity, RVS2 is recommended for estimating the merchantable volume in both species. This is relevant, given the importance of estimating precisely the merchantable volume of a tree profile. Furthermore, due to its simplicity, parsimony and ease of implementation, the proposed system of volume ratio equations is a possible option for forest managers as an efficient tool for calculating the distribution of timber products from standing trees and will help determine the volume and respective merchantable value for the two studied species in the forests of the *Pueblo Nuevo* region of *Durango*, Mexico. Systems based on volume ratio equations are simpler and easier to implement than the assessed segmented system.

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Conflict of interest

The authors declare that they have no conflict of interest. Dr. Gerónimo Quiñonez Barraza declares that he did not participate in any stage of the editorial process of this document.

Contribution by author

Francisco Cruz-Cobos, Gerónimo Quiñonez-Barraza, Verónica Hernández-Merino, Sacramento Corral-Rivas, and Adan Nava-Nava: data analysis, model fitting, and drafting and review of the manuscript; Francisco Cruz-Cobos and Gerónimo Quiñonez-Barraza: follow-up on the manuscript.

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