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Research article

Uso de guías para el manejo de la densidad en bosques mezclados de Michoacán

Use of density management diagrams for mixed stands in the state of *Michoacán*

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Abstract
Density guides (DG) are fundamental tools for silviculture, because they allow a better distribution of the growth space within the stand. The objective of this work was to construct and apply two DGs derived from a potential model (PM) and an exponential model (EM) for the thinning simulation. Two forest management objectives were tested: (1) To conserve, and (2) To change the proportion of species in the stand. Information from 1 346 minimum forest management units (MFMUs) with mixed species was used. The MFMUs were projected within the DGs in terms of the Stand Density Index (*SDI*), with respect to a maximum *SDI* (SDI_{max}). An MFMU was then randomly selected to deal with the aftereffects of thinning according to the two objectives aforementioned. The SDI_{max} were 883 (PM) and 800 (EM), on the basis of which the Langsaeter growth zones were defined as a percentage: 70 % imminent mortality, 40 % constant growth, and 20 % free growth. The thinning simulations indicated that in both 1 and 2, a greater tree and basimetric area removal was projected when using the PM DG than with the EM DG. The two DGs have implications for the final harvest projection due to the structure and definition of the *SDIs*.
Key words: Mixed forests, density index, exponential model, potential model, thinning simulation, growth zones.

Resumen
Las guías de densidad (GD) son herramientas fundamentales para la silvicultura porque permiten realizar una mejor distribución del espacio de crecimiento dentro del rodal. El objetivo de este trabajo fue construir y aplicar dos GD derivadas de un modelo potencial (MP) y uno exponencial (ME) para la simulación de aclareos. Se probaron dos objetivos de manejo forestal: (1) Para conservar, y (2) Para cambiar la proporción de especies del rodal. Se usó la información procedente de 1 346 unidades mínimas de manejo forestal (UMMF) con mezcla de especies. Las UMMF se proyectaron dentro de las GD en términos del Índice de Densidad de Rodal (*IDR*), con respecto a un *IDR* máximo (IDR_{max}). Después se seleccionó una UMMF de forma aleatoria para prescribir secuelas de aclareos de acuerdo con los dos objetivos planteados. Los IDR_{max} fueron de 883 (MP) y 800 (ME), con los cuales se definieron las zonas de crecimiento de *Langsaeter* en forma porcentual: 70 % mortalidad inminente, 40 % crecimiento constante y 20 % crecimiento libre. Las simulaciones de aclareos indicaron que, tanto en 1 como en 2, se proyectó una mayor remoción de árboles y de área basal cuando se

usó la GD del MP con respecto a la GD del ME. Las dos GD tienen implicaciones en la proyección de la cosecha final debido a la estructura y a la definición de los *IDR*.
Palabras clave: Bosques mezclados, índice de densidad, modelo exponencial, modelo potencial, simulación de aclareos, zonas de crecimiento.

Introduction

Density control considers a thinning schedule during stand growth and development (Torres-Rojo and Velázquez-Martínez, 2000). Density guides (DG) are useful forestry tools to simulate the crowding of trees on a site (Quiñonez-Barraza *et al.*, 2018) and project the stand dynamics (Cabrera-Pérez *et al.*, 2019). Newton (2021) classifies DGs into three groups: (1) Static dimensional DGs (density-size or size-density ratio); (2) Three-dimensional DG (density-size-time), and (3) *n*-dimensional DG (density-size-time-size-distribution).

The first DGs were based on the density-size relationship proposed by Reineke (1933), who defined maximum density by means of a straight line. The ratio gives a degree of occupancy in relative terms at the stand level, with respect to a maximum Stand Density Index (*SDI*) (Torres-Rojo and Velázquez-Martínez, 2000). Relative *SDIs* are used to compare the density between stands and to delimit the growth areas (GA) proposed by Langsaeter (Shaw, 2006).

Another mathematical expression was proposed by Quiñonez-Barraza and Ramírez-Maldonado (2019) for estimating the density/size ratio. The model is of the exponential type and projects a curved line to delimit the maximum density. Unlike in Reineke's model, the function estimates more realistic values for the intercept, which is associated with the initial density that occurs in the early stages of stand development.

Both the potential (PM) model of Reineke (1933) and the exponential model (EM) of Quiñonez-Barraza and Ramírez-Maldonado (2019) were used to estimate the maximum density line in mixed stands (Hernández-Martínez *et al.*, 2023) and to build DG in order to simulate thinning across Langsaeter's GAs (Quiñonez-Barraza and Ramírez-Maldonado, 2019).

In Mexico, several DGs have been developed for both pure and mixed stands (Quiñonez-Barraza *et al.*, 2018; Tamarit-Urias *et al.*, 2019). Most of the DGs generated have been carried out with coniferous species distributed in temperate climates (Cabrera-Pérez *et al.*, 2019), and less frequently in forest plantations of tropical species (García *et al.*, 1996; Minoche *et al.*, 2017).

Some examples of DG for Mexican natural coniferous forests are *Pinus patula* Schltdl. & Cham. (Santiago-García *et al.*, 2013; Camacho-Montoya *et al.*, 2018; Tamarit-Urias *et al.*, 2019), *Pinus rudis* Endl. (Martínez *et al.*, 2021), *Pinus teocote* Schltdl. & Cham. (Hernández *et al.*, 2013), *Pinus montezumae* Lamb. (Rodríguez *et al.*, 2009), *Pinus cooperi* C. E. Blanco var. *ornelasi* (Martínez) C. E. Blanco (Márquez-Linares and Álvarez-Zagoya, 1995), and *Pinus hartwegii* Lindl. (Zepeda and Villareal, 1987).

However, in forests with a mixture of Mexican species, there is little work on DGs. Corral-Rivas *et al.* (2015), Acevedo *et al.* (2018) and Cabrera-Pérez *et al.* (2019), used the Relative Spacing Index to construct DG in mixed stands for the north of the country. In the same region, Quiñonez-Barraza *et al.* (2018) and Quiñonez-Barraza and Ramírez-Maldonado (2019) created DGs using the Reineke Index (1933), while Martínez (2017) used both indices to develop DGs in mixed forests of *San Pedro El Alto* in *Zimatlán*, *Oaxaca* State.

The objective of this research study was to construct and apply two DGs to simulate thinning from two forest management approaches at the stand level: (1) Without considering the species composition and maintaining the current proportion of species, and (2) Consider the species composition and change their current proportion.

Materials and Methods

Study area

The study was carried out in the indigenous community of *Nuevo San Juan Parangaricutiro* (*CINSJP*), in the state of *Michoacán*, Mexico. It is located between 19°21′ and 19°34′ N and 102°08′ and 102°17′ W, with an average altitude of 2 550 m and a forest area of 9 914 ha. The climate is humid temperate, with different types of vegetation: coniferous, pine-oak, mountain mesophyll, and oak forests. The most economically important species are *Pinus pseudostrobus* Lindl., *Pinus montezumae*, *Abies religiosa* (Kunth) Schltdl. & Cham., *Quercus rugosa* Neé, and *Quercus laurina* Bonpl. (Hernández-Martínez *et al.*, 2023).

Data and construction of the DGs

Data from 9 559 temporary circular 1 000 m^2 sample plots distributed in 1 346 minimum forest management units (MFMU) in a natural forest, were utilized. In each plot, the tree diameter at 1.3 m height (*D*, cm) was measured with a model Inc. 800-647-5368 Jackson, MS Forestry Suppliers® diameter tape and the density (*Np*, number of trees per plot) was estimated. The *D* was used to obtain the individual basal area (*ba*, m^2). The *ba* and *Np* values were scaled to the hectare level; *BA*, m^2 ha^{-1} is the basal area, and *N* is the number of trees per hectare. In addition, the Quadratic mean diameter (*Dq*, cm) was calculated using Equation (1):

$$Dq = \sqrt{\frac{40\,000\,BA}{\pi\,N}} \qquad (1)$$

Where:

Dq = Quadratic mean diameter (cm)

BA = Basal area (m^2 ha^{-1})

π = 3.1416

In a previous study, the maximum density-to-size ratio (*MDTSR*) parameters were estimated for the PM and the EM (Hernández-Martínez *et al.*, 2023). The parameters were used in this work to define the line of maximum density with the equations (2) and (3).

$$N = e^{12.074} Dq^{-1.7096} \qquad (2)$$

$$N = e^{7.5933} e^{-0.0429Dq} \qquad (3)$$

Where:

N = Number of trees per hectare

Dq = Quadratic mean diameter (cm)

A reference *Dq* (Dq_r) of 25 cm was used to compare densities between stands (Reineke, 1933). The maximum *SDIs* (SDI_{max}) of the study area were 883 and 800 for the PM and EM, respectively. With these values, the relative *SDIs* of the MFMUs were calculated using Equation (4) for the PM and (5) for the EM.

$$SDI = N\left(\frac{Dq_R}{Dq}\right)^{\ddot{\beta}_1} \qquad (4)$$

$$SDI = Ne^{-\hat{\beta}_1 (Dq - Dq_R)} \qquad (5)$$

Where:

$\hat{\beta}_1$ = Estimated value of the slope for the PM and the EM, respectively

Equations (6) for PM and (7) for EM were used to estimate *N* based on a given *SDI*.

$$N = SDI\left(\frac{Dq}{Dq_R}\right)^{\hat{\beta}_1} \qquad (6)$$

$$N = SDIe^{\hat{\beta}_1 (Dq - Dq_R)} \qquad (7)$$

The isolines of the GAs were plotted according to the intervals proposed by Martínez (2017), and by Quiñonez-Barraza and Ramírez-Maldonado (2019). The limits were established with respect to the SDI_{max} for the abovementioned PM and EM. The beginning of the free GA was defined at 20 %, the lower limit of the constant GA at 40 %, and the beginning of the area of imminent mortality or self-thinning at 70 % of the SDI_{max}. The relative *SDI* was calculated for each of the MFMUs with the two models described above.

The constant GA was considered as the density optimum and is where the maximum growth potential is expressed before self-thinning (Santiago-García *et al.*, 2013). Thinning simulations were performed according to the methodology proposed by Martínez (2017), which is described in the following sections.

Simulation with constant species ratio

The objective is to obtain the i^{th} response (R_i) and the i^{th} growth projection (P_i) of a global thinning, where $i=1,...,n$, and n is the number of thinnings or the growth projection after thinning. Another objective is to evaluate the response and projection individually for the j^{th} species or group of species as appropriate, where $j=1,...,m$, and m is the number of species. When $i=n$ in the projection, P_i will be treated as the final harvest that is the result of the last thinning R_i.

To exemplify the above, we start from an initial condition (IC) in which the stand is in the self-thinning zone. Its values at stand level are N_0, BA_0, and Dq_0, and by species $N_{0,j}$, $BA_{0,j}$, and $Dq_{0,j}$, where j is the j^{th} species. The proportion by species [$Prop_{0,j}$ (%)] is calculated using the Equation (8).

$$Prop_{0,j}\,(\%) = \frac{N_{0,j}}{\sum_{j=1}^{m} N_{0,j}} \times 100 \qquad (8)$$

Where:

$N_{0,j}$ = Number of trees of the j^{th} species in the initial stand condition

When the stand exceeds the upper limit of the constant GA, a first thinning is applied to reduce the density to the lower limit of that GA. The results of the first overall thinning are as follows: N_{R1}, BA_{R1}, and Dq_{R1}, and, at the individual level: $N_{R1,j}$, $BA_{R1,j}$, and $Dq_{R1,j}$. Then, the stand undergoes an overall (N_{p1}, BA_{p1}, and Dq_{p1}) and an individual ($N_{p1,j}$, $BA_{p1,j}$, and $Dq_{p1,j}$) growth projection. The latest growth projection ($i=n$) is the final overall harvest for the N_{pn}, BA_{pn}, and Dq_{pn}, and of the final harvest by species ($N_{pn,j}$, $BA_{pn,j}$, and $Dq_{pn,j}$).

The procedure considers several steps to perform the simulation of thinning. The following sections describe in detail the steps and equations for two situations: thinning response and growth projection.

Thinning response

The overall number of residual trees (N_{Ri}) is estimated using Equation (9) for the PM, and Equation (10) for the EM. The Dq_0 is replaced with the Dq_{Pi-1} when i>1, and corresponds to the Dq of the growth prior to thinning i.

$$N_{R_i} = SDI_{LLCGA}\left(\frac{Dq_0}{Dq_R}\right)^{(\ddot{\beta}_1)} \qquad (9)$$

$$N_{R_i} = SDI_{LLCGA} \cdot e^{\ddot{\beta}_1 \cdot (Dq_0 - Dq_R)} \qquad (10)$$

Where:

SDI_{LLCGA} = SDI relative to the lower limit of the constant GA

The number of residual trees by species ($N_{Ri,j}$) is estimated based on its initial ratio [$Prop_{0,j}$ (%)], as shown in the expression (11).

$$N_{R_{i,j}} = N_{R_i} \times Prop_{0,j}\,(\%) \qquad (11)$$

The overall Dq_{Ri} is equal to Dq_0 when $i=1$, or Dq_{Pi-1}, which corresponds to the growth prior to thinning i when $i>1$. At the level of species, the $Dq_{Ri,j}$ will be equal to $Dq_{0,j}$ or $Dq_{Pi-1,j}$, the latter corresponds to the growth of the j^{th} species prior to thinning i.

The overall residual basal area after a thinning (BA_{Ri}) is estimated using Equation (12), and by species ($BA_{Ri,j}$), with Equation (13).

$$BA_{R_i} = \frac{\pi}{40\,000} \times Dq^2_{R_i} \times N_{R_i} \qquad (12)$$

$$BA_{R_{i,j}} = \frac{\pi}{40\,000} \times Dq^2_{R_{i,j}} \times N_{R_{i,j}} \qquad (13)$$

The overall cutting intensity (CI_{Ri}, %) is the difference between N_0 and N_{Ri} after thinning i when i=1, divided by N_0, as shown in expression (14). The value N_0 cannot be replaced by N_{P-1}, which indicates the number of trees projected prior to thinning i when i>1, as shown in Equation (15).

$$CI_{R_i}\,\% = \frac{N_0 - N_{R_i}}{N_0} \qquad (14)$$

$$CI_{R_i}\,\% = \frac{N_{P_{i-1}} - N_{R_i}}{N_{P_{i-1}}} \qquad (15)$$

Expressions (14) and (15) are applied in order to know the *CI* by species, which will be equal to the global CI_{Ri}, as the proportion of species is maintained in each thinning, without modifying the stand composition.

Projected growth

It is assumed that, after thinning *i*, the growth (P_i) will have the same number of residual trees from thinning, both overall and by species. Therefore, the following is true: $N_{Pi}=N_{Ri}$ and $N_{Pi,j}=N_{Ri,j}$.

The growth of *Dq* at stand level is obtained by clearing *Dq* in (6) for the PM and (7) for the EM, but expressed as a projection (Dq_{Pi}). In addition, the *SDI* value is replaced by the relative *SDI* of the upper limit of the constant GA (SDI_{ULCGA}). Therefore, the Dq_{Pi} are obtained with Equation (16) for the PM and (17) for the EM.

$$Dq_{P_i} = Dq_R \times \left(\frac{N_{P_i}}{SDI_{ULCGA}}\right)^{\left(1/\hat{\beta}_1\right)} \qquad (16)$$

$$Dq_{P_i} = \frac{ln\left(N_{P_i}/SDI_{ULCGA}\right)}{\hat{\beta}_1} + Dq_R \qquad (17)$$

Dq projection by species ($Dq_{Pi,j}$) is calculated using the expression (18), which applies to both models.

$$Dq_{P_{i,j}} = \frac{Dq_{P_i}}{Dq_0} \times Dq_{0,j} \qquad (18)$$

The overall projected basal area and by species is obtained with equations (12) and (13), respectively, expressed as BA_{Pi} and $BA_{Pi,j}$.

When obtaining the proportion by species with Equation (11) at this stage, the proportion is maintained according to the initial values of the stand.

Finally, when $i=n$ in the projection stage, the results obtained in the growth will refer to the final harvest.

The described steps are done iteratively until the number of thinnings n is reached. Analyses can be performed globally, without considering the species composition, as other authors have done in monospecific forests (Camacho-Montoya *et al.*, 2018; Tamarit-Urías *et al.*, 2019) and in mixed forests (Quiñonez-Barraza *et al.*, 2018; Quiñonez-Barraza and Ramírez-Maldonado, 2019).

Simulation with changes in the proportion of species

For the second management approach, the initial proportion of species will change at the end of the thinning simulation. Therefore, it is necessary to consider a taxon as a priority and its presence will be higher in terms of absolute density ratio (N); it will have higher values of N, Dq and BA in the final harvest. Species selection will depend on the needs of the forest owner, as well as on the management objective and production.

As in the previous approach, the starting point is an initial condition at the global level (N_0, BA_0, and Dq_0) and at the level of species ($N_{0,j}$, $BA_{0,j}$, $Dq_{0,j}$, and $Prop_{0,j}$ (%)). For some calculations, the initial condition is referred to as R_{i-1} or $R_{i-1,j}$ when i=1.

When i>1, it will correspond to the stand growth projection (P_{i-1}) or species ($P_{i-1,j}$) before the second or a later thinning. If the growth projection is i=n, it will correspond to the final harvest of the stand after the last thinning is applied. At the end of the thinning simulation, a stand dominated by the priority species and a species proportion different from the initial condition will be obtained.

Thinning response

When thinning is applied, the overall number of residual trees (N_{Ri}) is calculated with equations (9) and (10) according to the following model.

The $N_{Ri,j}$ will be a function of the cutting intensity (CI_{Ri}) that the forester applies to each species according to his criteria and experience, as long as a lower CI is applied to the priority species, compared to the rest of the taxa. Expression (19) is utilized to estimate the $N_{Ri,j}$.

$$N_{R_{i,j}} = N_{R_{i-1,j}} - \left(N_{R_{i-1,j}} \times CI_{R_{i,j}}\right) \qquad (19)$$

The $\sum_{j=1}^{m} N_{R_{i,j}} \sim N_{R_i}$ in order to reach the SDI_{ULCGA}.

It is assumed that $Dq_{Ri,j}$ is equal to $Dq_{Ri-1,j}$. If i=1, will be treated as the initial condition; when i>1 it will be the stand growth projection before or after the second thinning.

The basal area by species ($BA_{Ri,j}$) is obtained with Equation (13) and the overall basal area (BA_{Ri}) is calculated with Equation (20).

$$BA_{R_i} = \sum_{j=1}^{m} BA_{R_{i,j}} \qquad (20)$$

The Dq_{Si} is calculated based on the BA_{Ri} and N_{Ri}. Unlike in the previous approach, this will vary for each thinning, and is obtained with the expression (21).

$$Dq_{R_i} = \sqrt{\frac{40\,000}{\pi} \times \frac{BA_{R_i}}{N_{R_i}}} \qquad (21)$$

Projected growth

1. After thinning, it is assumed that the stand maintains the following equalities: $N_{Pi}=N_{Ri}$ and $N_{Pi,j}=N_{Ri,j}$, respectively.

Global Dq growth projection (Dq_{Pi}) is calculated with (16) and (17) of the first approach, according to the model being used.

Dq growth projection by species group ($Dq_{Pi,j}$) is similar to Equation (18) of the first approach, however, we add a quotient of $Dq_{Ri,j}$ and Dq_{Pi} additively as shown in expression (22).

$$Dq_{P_{i,j}} = \frac{Dq_{P_i}}{Dq_{R_i}} \times Dq_{R_{i,j}} + \frac{Dq_{R_{i,j}}}{Dq_{P_i}} \qquad (22)$$

The basal area of the growth projection by species $BA_{Ri,j}$ and overall BA_{Ri} is calculated using Equation (20) described above.

Finally, we obtain the proportion of species for each growth projection with the expression (11). If a higher *CI* is assigned to lower priority species, the stand will be dominated by taxa of greater economic importance, or of a particular purpose assigned by the forester.

The procedures described above have the advantage of implementing various strategies for simulating thinning (Martínez, 2017). According to the forest management approaches to be defined, pure or composite stands will be obtained with respect to the current stand conditions.

Results

Density guides

The SDI_{max} was 883 for the PM, and 800 for the EM. The relative *SDI* of the limits of the GA boundaries were, for the PM and the EM, as follows: (a) Start of self-thinning (*SDI*=618 and 560), (b) *LLCGA* (*SDI*=309 and 280), and (c) Beginning of the free GA (*SDI*=177 and 160). The *SDI* of the 1 349 MFMUs were projected in the DG (Figure 1). 0.5 % of the MFMU were found in the area of self-thinning (five MFMU with PM and seven with EM), while 21.8 % (294 MFMU) and 24.1 % (325 MFMU) were located within the constant GA; the rest were located below the upper limit of the free GA (Figure 1).

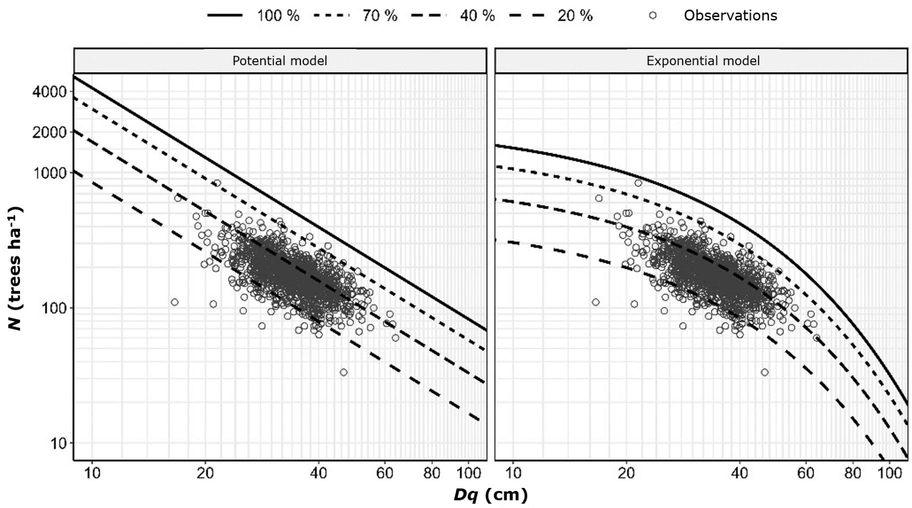


Figure 1. Density guides with the PM and the EM, with the observations recorded in the MFMUs.

In the thinning simulation, a MFMU located in the self-thinning area was selected. The selected MFMU had a surface area of 1 409 ha and consisted of *Pinus montezumae*, *Pinus lawsonii* Roezl *ex* Gordon, and *Abies religiosa*. Currently, the stand is under forest management with the Mexican Irregular Forest Management Method and the Regeneration by Selection Method. It has a relative *SDI* of 648 (73 % of the SDI_{max} of the PM) and 723 (90 % of the SDI_{max} of the EM). *Pinus montezumae* was considered as the priority species for the second approach. Initial stand condition data are shown in Table 1.

Table 1. Initial stand condition variables selected for thinning simulation.

Variable	***Pinus montezumae*** **Lamb.**	***Pinus lawsonii*** **Roezl *ex* Gordon**	***Abies religiosa*** **(Kunth) Schltdl. & Cham.**	**Total**
N_0 (trees ha^{-1})	730	10	100	840
BA_0 (m^2 ha^{-1})	24.85	0.18	5.44	30.46
Dq_0 (cm)	20.82	15	26.32	21.49

Prop₀ (%)	87	1	12	100

N_0 = Number of trees; BA_0 = Basal area; Dq_0 = Quadratic mean diameter; $Prop_0$ (%) = Ratio of the number of trees of the j^{th} species to the N_0.

Simulation of thinnings with two approaches to forest management

Three thinning operations were simulated in the selected MFMU. The results are shown in Table 2. The CI_{R1} applied in the first approach was higher in the EM (56 %) than in the PM (46 %). This is due to the curvature projected by the EM DG isolines, as well as to the asymptotic effect of the lines of the GA. After the second thinning, a CI_{Ri}>1 of 43 % was applied in both models and remained within the constant GA.

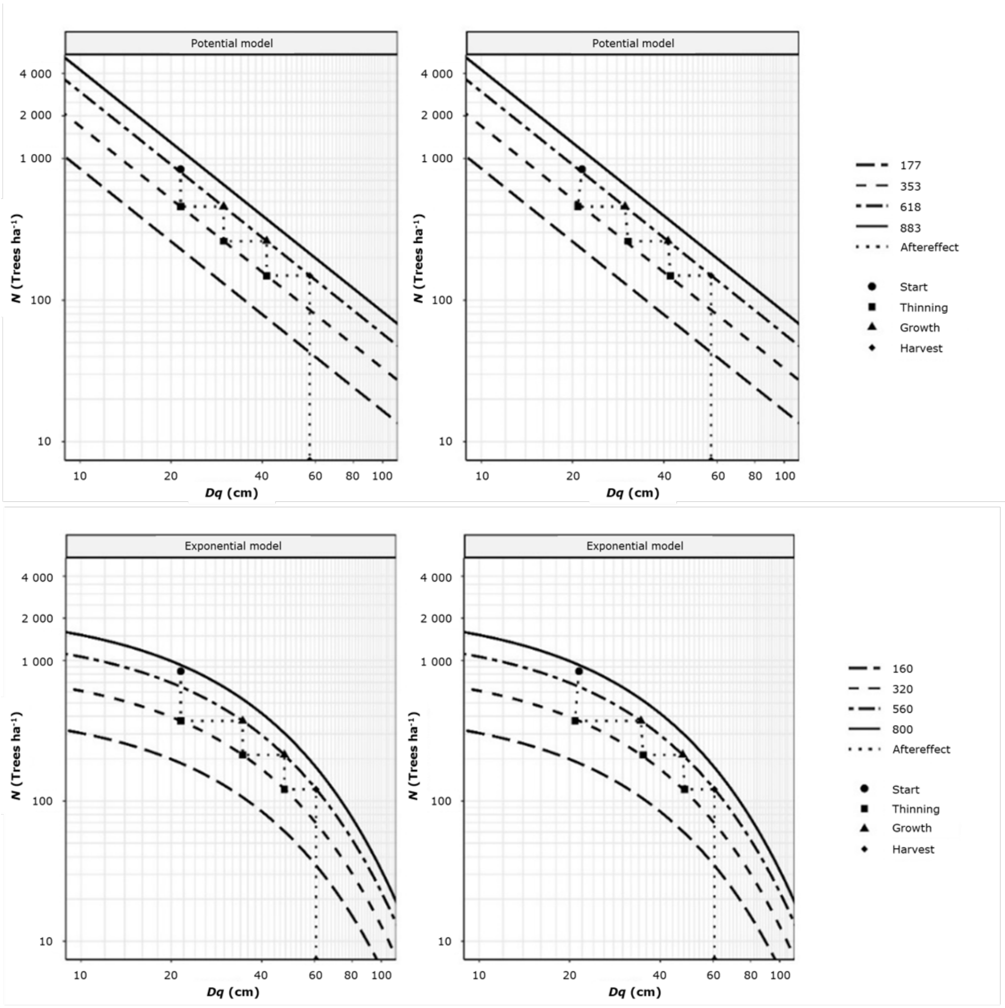
Table 2. Thinning aftereffects of a mixed stand with the PM and EM under two forest management objectives.

ST/RST	Variable	**O1: With constant species ratio**								**O2: With changes in the species ratio**							
		PM				**EM**				**PM**				**EM**			
		Pm (*j*=1)	**Pl (*j*=2)**	**Ar (*j*=3)**	**Overall total**	**Pm (*j*=1)**	**Pl (*j*=2)**	**Ar (*j*=3)**	**Overall total**	**Pm (*j*=1)**	**Pl (*j*=2)**	**Ar (*j*=3)**	**Overall total**	**Pm (*j*=1)**	**Pl (*j*=2)**	**Ar (*j*=3)**	**Overall total**
CI	N_0	730	10	100	840	730	10	100	840	730	10	100	840	730	10	100	840
	BA_0	24.85	0.18	5.44	30.46	24.85	0.18	5.44	30.46	24.85	0.18	5.44	30.46	24.85	0.18	5.44	30.46
	Dq_0	20.82	15	26.32	21.49	20.82	15	26.32	21.49	20.82	15	26.32	21.49	20.82	15	26.32	21.49
	$Prop_0$ (%)	87	1	12	100	87	1	12	100	87	1	12	100	87	1	12	100
R_1	N_{R1}	398	5	54	458	323	4	44	372	458	0	0	458	372	0	0	372
	BA_{R1}	13.53	0.10	2.96	16.59	11.00	0.08	2.41	13.49	15.58	0	0	15.58	12.67	0	0	12.67
	Dq_{R1}	20.82	15	26.32	21.49	20.82	15	26.32	21.49	20.82	0	0	20.82	20.82	0	0	20.83
	CI_{R1} (%)	46	46	46	46	56	56	56	56	37.3	100.0	100.0	45.5	49	100	100	55.7
P_1	N_{P1}	398	5	54	458	323	4	44	372	458	0	0	458	372	0	0	372
	BA_{P1}	26.05	0.19	5.70	31.93	28.43	0.2	6.22	34.85	33.45	0	0	33.45	36.08	0	0	36.08
	Dq_{P1}	28.88	20.81	36.51	29.81	33.46	24.11	42.29	34.54	30.5	0	0	29.81	35.13	0	0	34.54
	$Prop_{P1}$ (%)	87	1	12	100	87	1	12	100	100	0	0	100	100	0	0	100
R_2	N_{R2}	227	3	31	261	185	3	25	213	261	0	0	261	213	0	0	213
	BA_{R2}	14.88	0.11	3.26	18.25	16.24	0.12	3.56	19.91	19.10	0	0	19.1	20.6	0	0	20.6
	Dq_{R2}	28.88	20.81	36.51	29.81	33.46	24.11	42.29	34.54	30.50	0	0	30.5	35.13	0	0	35.13
	CI_{R2} (%)	43	43	43	43	43	43	43	43	42.9	0	0	42.9	42.9	0	0	42.9
P_2	N_{P2}	227	3	31	261	185	3	25	213	261	0	0	261	213	0	0	213
	BA_{P2}	28.64	0.2	6.27	35.12	30.83	0.22	6.75	37.8	36.38	0	0	36.38	38.99	0	0	38.99
	Dq_{P2}	40.07	28.87	50.65	41.36	46.10	33.22	58.27	47.58	42.09	0	0	41.36	48.32	0	0	47.58
	$Prop_{P2}$ (%)	87	1	12	100	87	1	12	100	100	0	0	100	100	0	0	100
R_3	N_{R3}	130	2	18	149	106	1	14	121	149	0	0	149	121	0	0	121
	BA_{R3}	16.37	0.12	3.58	20.07	17.62	0.13	3.86	21.6	20.79	0	0	20.79	22.26	0	0	22.26
	Dq_{R3}	40.07	28.87	50.65	41.36	46.10	33.22	58.27	47.58	42.09	0	0	42.1	48.32	0	0	48.3
	CI_{R3} (%)	43	43	43	43	43	43	43	43	42.9	0	0	42.9	42.9	0	0	42.9
P_3	N_{P3}	130	2	18	149	106	1	14	121	149	0	0	149	121	0	0	121
	BA_{P3}	31.50	0.22	6.9	38.62	28.61	0.2	6.26	35.07	39.61	0	0	39.61	36.00	0	0	36
	Dq_{P3}	55.58	40.05	70.26	57.37	58.74	42.32	74.25	60.63	58.10	0	0	57.37	61.45	0	0	60.63
	$Prop_{P3}$ (%)	87	1	12	100	87	1	12	100	100	0	0	100	100	0	0	100

O1 = Forest management objective 1; O2 = Forest management objective 2; ST/RST = Silvicultural treatment/response to the silvicultural treatment; *N* = Number of trees ha^{-1}; *BA* = Basal area (m^2 ha^{-1}); *Dq* = Quadratic mean diameter (cm); *Prop* = Proportion of *N* by species group (%); *IC* and suffix *0* = Initial condition of the stand; The suffix *R* = Thinning response (residual); The suffix *P* = Growth projection; Pm = *Pinus montezumae* Lamb.; Pl = *Pinus lawsonii* Roezl *ex* Gordon; Ar = *Abies religiosa* (Kunth) Schltdl. & Cham.

The values of the final harvest N_{P3}, BA_{P3}, and Dq_{P3} of the thinning simulation were similar in both models. The PM projected a larger basal area and more trees, whereas the EM projected a smaller basal area and fewer trees.

In the second management approach, *Pinus montezumae* was considered a priority species, and a higher *CI* was used for *Pinus lawsonii* and *Abies religiosa*. Due to the low density of *Pinus lawsonii* ($N_{0,2}$) and *Abies religiosa* ($N_{0,3}$), a 100 % *CI* was applied in the first thinning ($CI_{R1,2}$ and $CI_{R1,3}$, respectively) and only one *CI* of 37.7 % (PM) and 40 % (EM) for *Pinus montezumae* ($CI_{R1,1}$). The second and third thinning were only practiced on *Pinus montezumae* with a *CI* of 42.9 % in both models. The difference between this simulation and the previous one lies in obtaining a different Dq_{Ri} at each thinning. Therefore, $Dq_{Ri} \neq Dq_0$ and Dq_{Pi} (Table 2, Figure 2).



Approach 1: Upper and lower left panels. Approach 2: upper and lower right panels.

Figure 2. Density guides and thinning simulation with PM and EM.

The initial condition of the stands in the EM DG was close to the maximum density limit. In the DG for the PM, the initial condition was close to the lower limit of the self-thinning area. The differences are due to the shape of the isolines of the *SDIs* of the GAs of both DGs. The EM is more restrictive in the early stages of stand development, due to the asymptote present before 10 cm of *Dq*. The PM is more flexible, with a greater margin to reach the maximum density line in the early stages of stand development.

The *CIs* calculated in the two management approaches were close to 50 % of the initial density. A key factor of the *CI* in the first thinning is the initial condition of the stands; as long as the relative *SDI* of the stand is high and higher than the self-thinning, its *CI* will be higher. The *CI* of the thinnings after the first one remained fixed, as the simulation was programmed between the lower and upper limits of the constant GA.

The results indicate that using the PM DG, the final harvest projects a greater tree removal and a larger basal area than with the EM DG. Also, the *N*, *Dq* and *BA* were similar in the two models, according to the final harvest values shown in the first approach.

Discussion

This article presents a methodology and results of an alternative to simulate thinning in mixed stands, differentiating the species that make up the stand and their response in the dynamics of development and growth after a silvicultural intervention. In Mexico, DGs have been developed for several commercial species (Quiñonez-Barraza *et al.*, 2018), most of which have been built assuming monospecific and coetaneous stands (Camacho-Montoya *et al.*, 2018). This limits their use for managing the density of mixed forests because other tree species that are part of the site composition are not considered.

Other research has expanded Reineke's ratio to define the line of maximum density in stands with more than two species, and the expression generated thereby allows estimating specific intercept and slope parameters for each species or group of species (Torres-Rojo and Velázquez-Martínez, 2000). Thus, the equations proposed by Torres-Rojo and Velázquez-Martínez (2000) are another alternative for constructing DGs to those presented herein.

The approaches expressed in the analysis have been tested in very few papers. For example, Quiñonez-Barraza and Ramírez-Maldonado (2019) developed two DGs with the models presented in this research; and they point out that the EM is a more logical alternative than the PM, as the parameter $\widehat{\beta}_0$ is more realistic under natural conditions in the initial trajectory of the maximum density line. However, the PM tends to project very high estimates, perhaps because the maximum density line is drawn at a specific threshold of the density-size ratio (Hernández-Martínez *et al.*, 2023).

The relative *SDIs* are essential to define the GAs within the DGs, since they are associated with Langsaeter's theory to identify the status of the stand according to its degree of intra- and interspecific competition (Quiñonez-Barraza *et al.*, 2018). In addition, GAs are useful for achieving individual growth or biomass production maximization objectives (Santiago-García *et al.*, 2013), mainly in the constant GA. One of the advantages of the DGs presented herein is the ease of changing the *LLCGA* in equations (9) and (10) to reduce the *CI* and set a narrower range in the constant GA.

Different types of DGs have been discussed in other papers, resulting in a diversity of forms and designs, depending on the number and types of variables (Tamarit-Urias *et al.*, 2020). Prominent among them are the DGs proposed by Ginrich (1967), who uses the variables of *BA*, *N*, and *Dq* (Rodríguez *et al.*, 2009; Hernández *et al.*, 2013; Martínez *et al.*, 2021), as well as the DGs constructed through the Relative Spacing Index or the Hart-Becking Index using the variables *N*, dominant height, *Dq*, and volume (Corral-Rivas *et al.*, 2015; Acevedo *et al.*, 2018; Cabrera-Pérez *et al.*, 2019). Finally, there are the traditional DGs that use Reineke's ratio (Reineke, 1933) with a smaller number of variables for its construction, such as *N* and the *Dq*.

Tamarit-Urías *et al.* (2020) point out that a DG has to be developed with the minimum number of variables useful to the forester; those that include more than two variables are difficult to manipulate and interpret and are of little use (Quiñonez-Barraza *et al.*, 2018). On the other hand, specialized programs have also been created to encapsulate the complexity of mixed forests through computational systems to facilitate their use (Newton, 2021).

Density management in mixed forests still has areas for further development of knowledge and generation of forestry tools to facilitate decision making.

Conclusions

The density guidelines proposed in this research allow the forester to perform thinning simulations and final harvest projections, and determine felling intensities according to the current condition of a stand, based on management objectives and goals in mixed stands.

The density-size ratio (number of trees-mean square diameter) provides indices of stand density with respect to a maximum value, which is important for estimating stand occupancy in relative terms. The Relative Density Index is used to locate the stands within the density guidelines and their position in the Langsaeter growth areas in order to define strategies for future density management.

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Conflict of interest

The authors declare that they have no conflict of interest. Gerónimo Quiñonez-Barraza declares that he did not participate in the editorial process of this article.

Contribution by author

Abel Joseph Hernández-Martínez: data analysis, literature review, and drafting of the manuscript; Valentín José Reyes-Hernández: drafting and revision of the manuscript; Héctor Manuel de los Santos-Posadas: data analysis and drafting of the manuscript; Alejandro Velázquez-Martínez: review of the manuscript; Gerónimo Quiñonez-Barraza: data analysis and results.

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