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Research Article

Crecimiento en diámetro normal y área basal para *Pinus durangensis* Martínez en Madera, Chihuahua

Diameter at breast height and basal area growth for *Pinus durangensis* Martínez in *Madera*, *Chihuahua* State

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Abstract

Growth models are an efficient tool that helps to plan the application of silvicultural treatments in forest management. This study focused on fitting equations with fixed and random effects to predict the growth in diameter at breast height and basal area of *Pinus durangensis* in a forest community called *Cebadilla de Dolores ejido*, *Madera* municipality, state of *Chihuahua*, Mexico. Data from stem analysis of 27 trees were used (measured in 2021), obtaining slices at 1.30 m height and commercial measurements. Seven growth models were fitted, selecting the best one using 10 fitting statistics (*e. g.*, R^2, *RMSE*, *AIC*); in addition, the mixed-effects model procedure with fixed and random parameters at the individual tree level was used. The use of mixed-effects models provided estimates that allowed modeling the variability of diameters and basal area in terms of age. The results showed that age explained more than 98 % of the variability in diameter and basal area, with *RMSE* of 0.91 cm and 0.002 m^2, respectively. The Chapman-Richards model was the most effective for modeling the growth of the variables studied, with very efficient fitting statistics. In addition, mixed effects improved predictions at the individual tree level, incorporating fixed and random parameters.

Key words: Stem analysis, Chapman-Richards, growth, algebraic difference approach, mixed-effects models, fixed and random parameters.

Resumen

Los modelos de crecimiento son una herramienta eficiente que ayuda a planear la aplicación de los tratamientos silvícolas en el manejo de bosques. El objetivo del presente estudio fue ajustar ecuaciones con efectos fijos y aleatorios para predecir el crecimiento en diámetro normal y área basal de *Pinus durangensis* en el ejido Cebadilla de Dolores, municipio Madera, Chihuahua, México. Para ello se utilizaron datos de análisis troncales de 27 árboles medidos en 2021, de los que se obtuvieron rodajas a 1.30 m de altura y medidas comerciales. Se ajustaron siete modelos de crecimiento, y se seleccionó el mejor mediante 10 estadísticos de ajuste (*e. g.*, R^2, *RMSE*, *AIC*); además, se siguió el procedimiento de modelos de efectos mixtos con parámetros fijos y aleatorios a nivel de árbol individual. El uso de modelos de efectos mixtos proporcionó estimaciones que permitieron modelar la variabilidad de los diámetros y el área basal con respecto a la edad. Los resultados indican que la edad explicó más de 98 % de la variabilidad del diámetro normal y del área basal, con *RMSE* de 0.91 cm y 0.002 m^2, respectivamente. El modelo *Chapman-Richards* fue el más efectivo para modelar el crecimiento de las variables estudiadas, con estadísticas de ajuste muy eficientes. Además, los efectos mixtos mejoraron las predicciones a nivel de árbol individual, al incorporar parámetros fijos y aleatorios.

Palabras clave: Análisis troncales, *Chapman-Richards*, crecimiento, diferencia algebraica, modelos de efectos mixtos, parámetros fijos y aleatorios.

Introduction

Growth models facilitate decision-making and the planning of forestry treatments in forest management (von Gadow et al., 2004). Growth can be measured at the individual tree or stand level, using diameter, basal area, height, volume and biomass, stand quality, and density and competition indicators (Monserud, 2003). Growth information is important for estimating rotation age and treatment intensity based on the forest's productive potential (Hasenauer, 2006).

Over time, several forest growth models have been developed, including individual-tree and stand-based ones. The latter are the most appropriate for even-aged and regular stands, as they are difficult to apply in complex stands due to the diversity of species, ages, heights, and diameters (Porté & Bartelink, 2002; Vanclay, 1994). Individual-tree models are applicable to both pure and mixed forests. Growth prediction at the stand level is facilitated by summing the individual growth of each species (Hernández et al., 2021).

Regardless of whether growth is modeled at the individual tree or stand level, attempts have been made to increase the accuracy of model fitting by applying various techniques such as Algebraic difference (ADA), Generalized algebraic difference (GADA), and Mixed-effects models (MEM) (Bailey & Clutter, 1974; Castillo-López et al., 2018; Corral et al., 2019).

Mixed-effects models allow for the analysis of data with repeated measurements, considering both types of variability existing in longitudinal data and the nonlinear relationship between the response variable and time. They are flexible in representing heterogeneity and handling incomplete and unbalanced data, which are common in longitudinal data and trunk analyses of mixed forests (García et al., 2013), and provide consistent estimates of fixed parameters and their standard errors (Castedo et al., 2006), as well as the definition of random parameters at the tree, stand, or ecological region level.

The inclusion of random parameters, which are specific to each sampling unit, allows for modeling the variability of a given phenomenon across different locations after defining a common fixed functional structure (Lindstrom & Bates, 1990). Furthermore, they can improve predictions if the value of the random parameters can be estimated for an unsampled location through a random parameter calibration process. This approach is known as localization or calibration and can be applied if complementary observations of the dependent variable are available, in this case, the diameter at breast height and basal area of individual trees (Castedo et al., 2006).

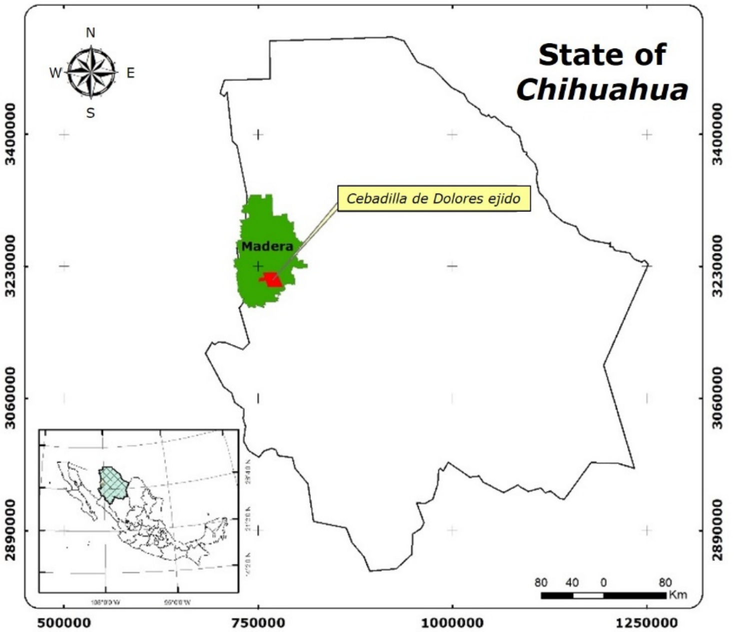
Understanding forest diameter and basal area growth is essential for analyzing their dynamics and evaluating forestry alternatives (Donoso et al., 2018). Developing growth functions in mixed-species forests improves the accuracy of estimates and helps reduce inventory time and costs by measuring a subsample in the field (Guerra-De la Cruz et al., 2019).

Based on the above, the objective of this study was to fit growth equations to describe the diameter at breast height-age and basal area-age relationships using Mixed-effects models for *Pinus durangensis* Martínez in a forest in *Madera*, *Chihuahua*, Mexico.

Materials and Methods

Study area

The study was conducted in several temperate forests in the *Cebadilla de Dolores ejido*, located in the *Sierra Madre Occidental* mountain range, 56 km Southwest of *Madera* municipality, in the state of *Chihuahua* (Figure 1). The property covers 38 442.77 ha, of which 4 895.22 ha are forested land and 4 573.15 ha are used for timber harvesting. The climate is temperate subhumid with summer rainfall, and the altitude ranges from 1 175 to 2 510 m (García, 2004). The most important plant communities are made up of mixed forests with species of the *Pinus* and *Quercus* genera, in addition to some shrub species (Consultoría Ambiental Agropecuaria Forestal y Financiera [CAAFF], 2015).



Madera = *Madera* municipality.

Figure 1. Geographic location of the *Cebadilla de Dolores ejido*, *Madera* municipality, State of *Chihuahua*, Mexico.

Used Variables

The variables used were diameter at breast height and basal area, which were related to age. To estimate the diameter at breast height, the radii of each slice were measured with a Nobrand® ruler graduated in mm and multiplied by two (Equation 1). The basal areas were obtained from the diameter at breast height (Equation 2) (Buendía-Rodríguez et al., 2019).

$$dn_i = 2r \qquad (1)$$

$$ab_i = \frac{\pi}{40\,000}\,(dn_i)^2 \qquad (2)$$

Where:

dn_i = Diameter at breast height at age *i* (cm)

r = Radius of the sections

ab_i = Basal area at age *i* (m^2)

Data

The data were collected from 27 trees using selective and destructive sampling under different site conditions. Slices were obtained from each tree at 1.30 m from the base using a model MS 172 STIHL®, 3/8"P chainsaw and the tree number was labeled as control information (Klepac, 1983). In addition, the number of rings was counted from the center to the periphery, generating a database of diameter at breast height for all ages. Because stem analysis is a destructive sampling method, only 27 trees were considered due to their disposition for felling. However, by using the growth time series, enough information was obtained to model the growth of the diameter at breast height and the basal area.

Used Models

To describe the growth in diameter at breast height and basal area of *Pinus durangensis*, based on the growth rings of the 1.3 m log, a first fit was performed with seven growth models using the Generalized nonlinear modeling technique. The structure of the models used (equations 3 to 9) was adopted by Zeide (1993):

M1, Chapman-Richards: $y_{ij} = \alpha_0(1 - e^{-\alpha_1 t_{ij}})^{\alpha_2} + \varepsilon_{ij}$ (3)

M2, Hossfeld IV: $y_{ij} = \left(\frac{t_{ij}^{\alpha_2}}{(\alpha_1 + \frac{t_{ij}^{\alpha_2}}{\alpha_0})}\right) + \varepsilon_{ij}$ (4)

M3, Gompertz: $y_{ij} = \alpha_0 \, e^{-\alpha_1 \, e^{-\alpha_2 t_{ij}}} + \varepsilon_{ij}$ (5)

M4, Weibull: $y_{ij} = \alpha_0(1 - e^{-\alpha_1 t_{ij}^{\alpha_2}}) + \varepsilon_{ij}$ (6)

M5, Logistic: $y_{ij} = \left(\frac{\alpha_0}{(1+\alpha_2 \, e^{-\alpha_1 t_{ij}})}\right) + \varepsilon_{ij}$ (7)

M6, Monomolecular: $y_{ij} = \alpha_0(1 - \alpha_2 \, e^{-\alpha_1 t_{ij}}) + \varepsilon_{ij}$ (8)

M7, Korf: $y_{ij} = \alpha_0 \, e^{-\alpha_1 t_{ij}^{-\alpha_2}} + \varepsilon_{ij}$ (9)

Where:

y_{ij} = Diameter at breast height or j basal area in the *i* tree

α_0, α_1, α_2 = Parameters to estimate

e = Exponential function

t_{ij} = j age in the *i* tree

ε_{ij} = j random error in the *i* tree

Since the ages of the trees undergoing trunk analysis ranged from 29 to 67 years, the diameters and basal areas of all trees were projected to the maximum recorded age, in order to obtain a symmetrical dataset and for the modeling to be based on the same range of the dependent variable as a balanced time series (Harvey & Shephard, 1993; Parzen, 1961). The projection of each of the aforementioned variables was carried out using the anamorphic and polymorphic models in algebraic differences and derivatives of the Chapman-Richards model (M1; Equation 3) (Richards, 1959), since in a preliminary analysis it was the most efficient, according to the fit statistics used (Equation 10 and 11):

M8, Chapman-Richards-Anamorphic: $y_2 = y_1 \left(\frac{1-e^{-\alpha_1 t2}}{1-e^{-\alpha_1 t1}}\right)^{\alpha_2}$ (10)

M9, Chapman-Richards-Polymorphic: $y_2 = \alpha_0 \left(\frac{y_1}{\alpha_0}\right)^{\frac{ln(1-e^{-\alpha_1 t_2})}{ln(1-e^{-\alpha_1 t_1})}}$ (11)

Where:

α_0, α_1, α_2 = Parameters to estimate

e = Exponential function

y_1, y_2 = Diameter at breast height or basal area in states 1 and 2

t_1 , t_2 = Age in states 1 and 2

ln = Natural logarithm

Subsequently, the model fitted with the most efficient fit statistics of the ranking system (Kozak & Smith, 1993) was reparameterized with fixed and random effects, in order to represent the variability of growth in diameter at breast height and basal area of the tree through nonlinear models with mixed effects (Pinheiro & Bates, 2006; Pinheiro et al., 2009). Mixed models are recommended when there are correlated data as in this case.

Using the most efficient baseline model in a preliminary and comparative analysis (ranking system statistics), three nonlinear mixed-effects models (MEMs) were formulated based on the formulation of nonlinear models with fixed and random effects (Pinheiro et al., 2009). In the first model, only the random effect of the asymptote parameter "α_0" was considered (M10; Equation 12), while in the second, the random effects of the asymptote parameter combined with the shape parameter "α_0, α_1" were considered (M11; Equation 13), and in the third, the random effects of the asymptote parameter combined with the shape parameter "α_0, α_2" (M12; Equation 14).

$$\text{M10: } y_{ij} = (\alpha_0 + \alpha_{0i})(1 - e^{-\alpha_1 t_{ij}})^{\alpha_2} + \varepsilon_{ij} \qquad (12)$$

$$\text{M11: } y_{ij} = (\alpha_0 + \alpha_{0i})(1 - e^{(-\alpha_1 + \alpha_{1i})t_{ij}})^{\alpha_2} + \varepsilon_{ij} \qquad (13)$$

$$\text{M12: } y_{ij} = (\alpha_0 + \alpha_{0i})(1 - e^{-\alpha_1 t_{ij}})^{(\alpha_2 + \alpha_{2i})} + \varepsilon_{ij} \qquad (14)$$

Where:

y_{ij} = Diameter at breast height or basal area *j* in the *i* tree

α_0, α_1 and α_2 = Fixed parameters

e = Exponential function

t_{ij} = Age *j* in the *i* tree

α_{0i}, α_{1i}, α_{2i} = Random parameters at the tree level

ε_{ij} = Random error *j* in the *i* tree

The nonlinear mixed-effects models considered the error term according to Pinheiro and Bates (2006), and the unit of study was the tree. For each variable, the generic formulation was as follows (Equation 15) (Corral et al., 2019):

$$y_{ij} = f\left(t_{ij}, \varphi_i\right) + \varepsilon_{ij}, \varepsilon_{ij} \sim N(0, \sigma^2), \emptyset_i = A_i\lambda + B_i b_i \qquad (15)$$

Where:

y_{ij} = Diameter at breast height or basal area *j* in the *i* tree

f = *1*×*p* covariate growth model function (Richards, 1959)

φ_i = Parameter vector for the *i* tree

ε_{ij} = Error term, assumed to be independent and normally distributed with zero mean and σ^2 variance

$\emptyset_i$ = $r \times 1$ parameter vector (r is the number of parameters in the model) specific to the *i* tree

λ = $p \times 1$ vector of fixed parameters

b_i = Vector of random parameters associated with the *i* tree

A_i, B_i = $r \times p$ and $r \times q$ design matrices for the fixed and random effects specific to the *i* tree, respectively

Model fitting

Parameter estimation for the base models was performed using generalized linear models in the "gnls" package of R software version 4.3.3 (R Core Team, 2024), while the MEMs were fitted using maximum likelihood in the "nlme" package of R software version 4.3.3 (R Core Team, 2024) using the algorithms developed by Pinheiro and Bates (2006). These procedures have been successfully studied in height-diameter models (Corral et al., 2019).

Fit statistics

The fitting performance of the models was measured using the Adjusted coefficient of determination (R^2), the Fit index (*FI*), the Root mean square error (*RMSE*), the

Standard error of the estimate (*SEE*), the Relative standard error of the estimate (*RSEE*), the Mean error (*E*), the Relative mean error (*RE*), the Akaike information criterion (*AIC*), the Bayesian information criterion (*BIC*), and the log likelihood (*LogLik*). These statistics have been successfully used in systems of taper and volume equations (Quiñonez-Barraza et al., 2019; Zhao et al., 2018). In addition, the base models were evaluated using a ranking system with the statistics proposed by Kozak and Smith (1993) and applied by Quiñonez-Barraza et al. (2019) in the evaluation of taper and volume equations.

Test normality of random effects

The normality of the random parameter estimates was assessed using Shapiro-Wilk normality tests (Hanusz et al., 2016; Shapiro & Wilk, 1965) at a significance level of 1 %. This is consistent with the theory of MEM (Pinheiro et al., 2009).

Results and Discussion

Growth trend projection

Data projections and symmetry of growth trends were performed using the base model M1 (Equation 3), with the anamorphic (M8; Equation 10) and polymorphic (M9; Equation 11) equations. Although both models showed acceptable projections, the polymorphic model (M9; Equation 11) generated better representations for the two variables studied. This model assumes that growth rates at different tree sites were different and is related to the bases of algebraic difference equations (Bailey & Clutter, 1974).

Figure 2 illustrates the trends for diameter at breast height and basal area. The generation of asymmetric datasets offered the possibility of modeling growth for a balanced and symmetric time series (Harvey & Shephard, 1993). The trends showed the growth rates in the study variables. The findings are similar to those reported by Tamarit-Urias et al. (2021) on diameter growth for *Pinus montezumae* Lamb. in forests of the state of *Puebla*, Mexico.

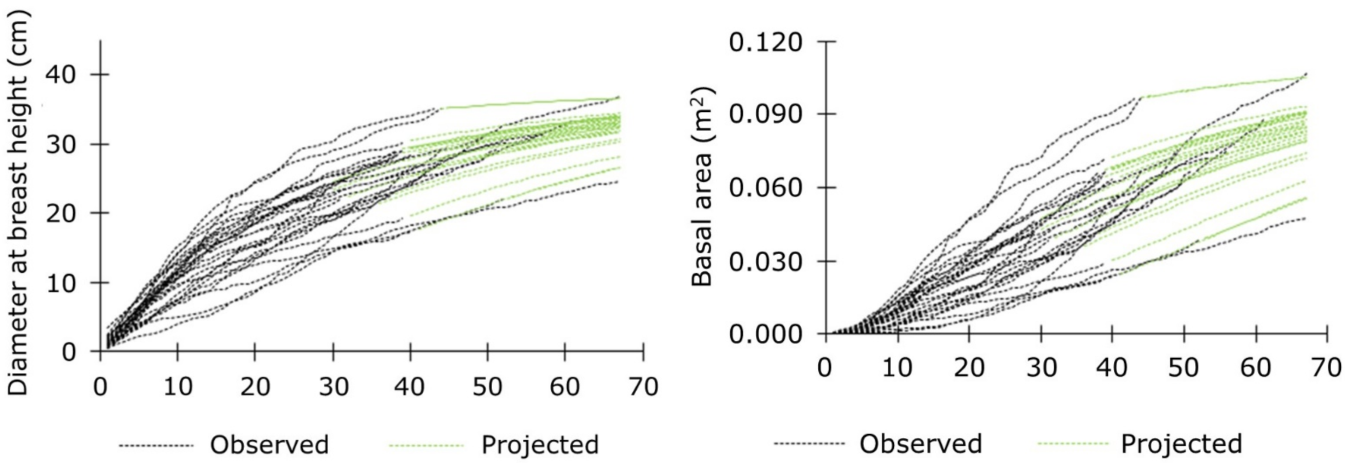


Figure 2. Projection of growth trajectories for diameter at breast height and basal area at age 67 using the anamorphic and polymorphic equations of the M1 model.

Diameter at breast height-age and basal area-age relationships

Table 1 lists the estimated parameters for the seven growth models fitted to the diameter at breast height-age and basal area-age relationships. It can be seen that all parameters were significantly different from zero at the 1 % significance level, for both diameter at breast height and basal area. Therefore, the models studied by Zeide (1993) and used in other research performed by Aguirre-Bravo and Smith (1986) and Quiñonez et al. (2015) satisfactorily modeled growth in diameter at breast height and basal area for *Pinus durangensis.*

Table 1. Parameter estimators for the models fitted for diameter at breast height and basal area.

M	Par.	Diameter at breast height				Basal area			
		Est.	***E. E.***	***t*-val**	***p*-value**	**Est.**	***E. E.***	***t*-val**	***p*-value**
M1	α_0	34.1	1.436	23.79	<0.00001	0.097	0.009	10.3	<0.00001
	α_1	0.03	0.003	8.20	<0.00001	0.030	0.005	5.81	<0.00001
	α_2	0.94	0.053	17.71	<0.00001	1.768	0.170	10.3	<0.00001
M2	α_0	45.3	3.487	13.00	<0.00001	0.116	0.014	8.02	<0.00001
	α_1	0.85	0.111	7.62	<0.00001	4 344.7	0.142	3.05	0.00229
	α_2	1.05	0.059	17.69	<0.00001	1.628	0.115	14.0	<0.00001
M3	α_0	29.1	0.491	59.38	<0.00001	0.078	0.003	24.3	<0.00001
	α_1	2.28	0.069	32.81	<0.00001	3.795	0.186	20.3	<0.00001
	α_2	-0.07	0.003	-23.78	<0.00001	-0.056	0.003	15.9	<0.00001
M4	α_0	34.5	1.779	19.42	<0.00001	0.086	0.007	11.4	<0.00001
	α_1	0.03	0.002	14.19	<0.00001	0.003	0.000	5.25	<0.00001
	α_2	0.95	0.041	23.32	<0.00001	1.516	0.084	18.0	<0.00001
M5	α_0	27.7	0.379	73.12	<0.00001	0.068	0.001	36.3	<0.00001
	α_1	0.10	0.003	26.96	<0.00001	0.102	0.004	20.9	<0.00001

	α_2	5.68	0.300	18.93	<0.00001	16.59	1.560	10.6	<0.00001
M6	α_0	33.4	0.937	35.72	<0.00001	0.221	0.058	3.75	0.00017
	α_1	0.03	0.002	16.42	<0.00001	0.007	0.002	3.19	0.00142
	α_2	0.99	0.011	86.11	<0.00001	1.023	0.010	99.8	<0.00001
M7	α_0	99.5	22.67	4.38	<0.00001	0.371	0.164	2.25	0.02420
	α_1	5.11	0.118	43.20	<0.00001	10.79	1.172	9.20	<0.00001
	α_2	0.35	0.046	7.59	<0.00001	0.460	0.089	5.16	<0.00001

M = Model; Par. = Parameter; Est. = Estimator; *E. E.* = Standard Error; *t*-val = *t*-value; α_0, α_1 and α_2 = Fixed parameters.

Table 2 shows the fit statistics and the score obtained for each of the growth models in diameter at breast height and basal area in their basic form, in which the lowest score corresponds to the model with the best performance (Kozak & Smith, 1993). Based on the above, the Weibull model (M4; *C*=25) was the one with the best fitting in diameter, followed by the Chapman-Richards (M1; *C*=28), Hossfeld IV (M2; *C*=30), Logistic (M5; *C*=36), Gompertz (M3; *C*=40), Monomolecular (M6; *C*=59), and Korf (M7; *C*=61) models. Regardless of the better fit of the Weibull model, it was decided, given its flexibility and wide use, to apply the M1 model to represent the estimates of diameter at breast height.

Table 2. Fit statistics for diameter at breast height and basal area.

					Diameter at breast height						
M	***R^2***	***FI***	***RMSE***	***SEE***	***RSEE***	***E***	***RE***	***AIC***	***BIC***	***LogLik***	**C**
M1	0.80	0.80	3.83	3.83	22.17	-0.02	-0.11	6 617	6 637	-3 304	39
M2	0.80	0.80	3.83	3.84	22.14	-0.01	-0.06	6 613	6 633	-3 302	28
M3	0.80	0.80	3.83	3.84	22.16	-0.02	-0.12	6 616	6 636	-3 304	38
M4	0.80	0.80	3.82	3.83	22.12	0.009	0.05	6 611	6 631	-3 301	18

M5	0.80	0.80	3.84	3.83	22.21	$-1x10^{-7}$	$-7x10^{-7}$	6 621	6 641	-3 306	37
M6	0.79	0.79	3.93	3.93	22.75	-0.04	-0.28	6 678	6 698	-3 335	56
M7	0.78	0.78	4.03	4.03	23.32	-0.08	-0.51	6 738	6 758	-3 365	64

Basal area

M	R^2	***FI***	***RMSE***	***SEE***	***RSEE***	***E***	***RE***	***AIC***	***BIC***	***LogLik***	***C***
M1	0.71	0.71	0.01	0.01	43.66	$-3x10^{-4}$	-0.10	-7 013	-6 992	3 510	24
M2	0.71	0.71	0.01	0.01	43.66	$-3x10^{-4}$	-0.12	-7 013	-6 992	3 510	30
M3	0.71	0.71	0.01	0.01	43.67	$-7x10^{-4}$	-0.24	-7 012	-6 992	3 510	38
M4	0.71	0.71	0.01	0.01	43.67	$4x10^{4}$	0.16	-7 012	-6 992	3 510	40
M5	0.70	0.71	0.01	0.01	43.85	$2x10^{-9}$	$1x10^{-5}$	-7 003	-6 982	3 505	42
M6	0.70	0.71	0.01	0.01	43.82	$-1x10^{-4}$	-0.57	-7 004	-6 984	3 506	48
M7	0.70	0.70	0.01	0.01	44.31	$-3x10^{-4}$	-1.23	-6 978	-6 957	3 493	58

M = Model; R^2 = Adjusted coefficient of determination; *FI* = Fit index; *RMSE* = Root mean square error; *SEE* = Standard error of estimate; *RSEE* = Relative standard error of estimate; *E* = Absolute mean error; *RE* = Relative mean error; *AIC* = Akaike information criterion; *BIC* = Bayesian information criterion; *LogLik* = Logarithm of the likelihood; *C* = Model score.

On the other hand, the Chapman-Richards model (M1; *C*=24) had the best fit in basal area, followed by the Hossfeld IV (M2; *C*=30), Gompertz (M3; *C*=38), Weibull (M4; *C*=40), Logistic (M5; *C*=42), Monomolecular (M6; *C*=48), and Korf (M7; *C*=58) models. Model M1 was also used to represent basal area estimates.

The results of this study are similar to those reported by Hernández et al. (2020), who evaluated growth models for diameter at breast height, basal area, total height and stem volume using the Chapman-Richards, Schumacher, Hossfeld I and Weibull models for individual *Pinus pseudostrobus* Lindl. and *Pinus oocarpa* Schiede *ex* Schltdl trees in the state of *Guerrero*, Mexico. In their research, they identified the Chapman-Richards model as the best for estimating the basal area of *Pinus oocarpa*, with an R^2 of 0.9154 and an *RMSE* of 0.0096 m^2. In a similar way, Corral and Návar (2005) fitted growth and diameter at breast height increase equations for *Pinus* species in the *El Salto* region, *Durango* state, Mexico; they concluded that the Chapman-Richards model showed the greatest growth and increments at the tree group level and individually for *Pinus durangensis*, with an R^2 of 0.70 and *RMSE* of 3.29 cm.

On the other hand, Domínguez-Calleros et al. (2017) adjusted the Log Normal, Chapman-Richards, von Bertalanfy, Logistic and Gompertz growth models, also based on Zeide's (1993) formulations, with time-integrable equations. They determined that the Chapman-Richards model provided the best fit, although they concluded that further research is needed to review the limits within which the assumptions apply when fitting models to time series.

The results of the study described here also agree with those presented by Ramirez et al. (2014), who compared diameter at breast height growth models for *Eucalyptus urophylla* S. T. Blake, used data from 55 trees and concluded that the Chapman-Richards growth model had a lower standard error; based on the *AIC*, they determined that this model was the best.

Diameter at breast height-age and basal area-age relationships with mixed effects

Table 3 lists the parameter estimators and statistical properties for the three combinations of mixed-effects growth models fitted to the diameter at breast height-age and basal area-age relationships, based on the formulation of nonlinear mixed-effects models (Corral et al., 2019; Pinheiro et al., 2009). In this case, all estimated parameters were significantly different from zero at a significance level of 1 %. The parameters associated with the variance of the random effects and the error term for the combinations of mixed parameters and study variables are also shown.

Table 3. Parameter estimators for the mixed-effects models for diameter at breast height and basal area.

M	Par.	Diameter at breast height				Basal area			
		Est.	*E. E.*	*t*-val	*p-value*	Est.	*E. E.*	*t*-val	*p*-value
M10	α_0	37.66	1.104	34.1	<0.00001	0.107	0.005	19.5	<0.00001
	α_1	0.027	0.001	24.0	<0.00001	0.029	0.001	22.1	<0.00001
	α_2	0.913	0.018	49.6	<0.00001	1.814	0.053	34.0	<0.00001
	$sd(\alpha_{0i})$	5.150	0.124	41.3	<0.00001	0.025	0.006	41.8	<0.00001
	σ	1.726	0.041	41.3	<0.00001	0.005	0.001	53.0	<0.00001
M11	α_0	40.60	1.586	25.5	<0.00001	0.128	0.007	17.5	<0.00001
	α_1	0.028	0.002	12.2	<0.00001	0.029	0.002	12.3	<0.00001
	α_2	0.948	0.009	104.1	<0.00001	1.952	0.023	83.8	<0.00001
	$sd(\alpha_{0i})$	7.811	0.189	41.3	<0.00001	0.035	0.008	44.7	<0.00001
	$sd(\alpha_{1i})$	0.011	0.002	58.0	<0.00001	0.011	0.002	58.5	<0.00001
	σ	0.854	0.020	41.4	<0.00001	0.002	0.001	22.0	<0.00001
M12	α_0	37.19	0.976	38.0	<0.00001	0.105	0.004	23.3	<0.00001
	α_1	0.030	0.006	50.0	<0.00001	0.034	0.006	55.3	<0.00001
	α_2	1.030	0.059	17.4	<0.00001	2.325	0.149	15.5	<0.00001
	$sd(\alpha_{0i})$	4.926	0.119	41.3	<0.00001	0.022	0.005	45.6	<0.00001
	$sd(\alpha_{2i})$	0.301	0.007	41.8	<0.00001	0.755	0.018	41.5	<0.00001
	σ	0.929	0.022	41.4	<0.00001	0.002	0.001	24.0	<0.00001

M = Model; Par. = Parameter; Est. = Estimator; *E. E.* = Standard Error; *t*-val = *t*-value; α_0, α_1 and α_2 = Fixed parameters; α_{0i}, α_{1i} and α_{2i} = Random parameter estimators; *sd* = Standard deviation of the random parameter; σ = Model variance.

Table 4 summarizes the fitting statistics for the mixed-effects models for predicting growth in diameter and basal area. Overall, the three models fitted to the species showed satisfactory statistical goodness of fit, although the best explanation resulted from the fit of the M12 model (Equation 14) by associating the fixed parameters α_0 and α_2 with the random parameters α_{0i} and α_{2i} additively ($\alpha_0 + \alpha_{0i}$) and ($\alpha_2 + \alpha_{2i}$) to predict growth in diameter at breast height and basal area as a function of age, respectively. These statistics demonstrated that the mixed-effects models are superior to models with only fixed effects (Corral et al., 2019). They are better because they consider variability within and between trees.

Table 4. Fitting statistics of the mixed-effects models for the diameter at breast height and basal area variables.

Diameter at breast height										
M	***R²***	***FI***	***RMSE***	***SEE***	***RSEE***	***E***	***RE***	***AIC***	***BIC***	***LogLik***
M10	0.96	0.96	1.71	1.72	8.02	-0.13	-0.02	7 265	7 292	-3 627
M11	0.97	0.97	1.36	1.38	6.45	-0.39	-0.08	4 879	4 918	-2 432
M12	0.99	0.99	0.91	0.93	4.32	0.01	0.07	5 204	5 243	-2 595
Basal area										
M	***R²***	***FI***	***RMSE***	***SEE***	***RSEE***	***E***	***RE***	***AIC***	***BIC***	***LogLik***
M10	0.96	0.96	0.005	0.005	12.26	0.002	-0.41	-13 644	-13 617	6 827
M11	0.98	0.98	0.003	0.003	8.15	0.002	-0.46	-16 610	-16 572	8 312
M12	0.99	0.99	0.002	0.002	5.73	0.001	0.27	-16 206	-16 167	8 110

R^2 = Adjusted coefficient of determination; *FI* = Fit index; *RMSE* = Root mean square error; *SEE* = Standard error of estimate; *RSEE* = Relative standard error of estimate; *E* = Absolute mean error; *RE* = Relative mean error; *AIC* = Akaike information criterion; *BIC* = Bayesian information criterion; *LogLik* = Log likelihood.

Figure 3 shows the graphical behavior of estimates and observed data for diameter at breast height and basal area *versus* age for each combination of mixed-effects model fitting. By including random effects, variability in growth series at the individual tree level is assumed in all cases.

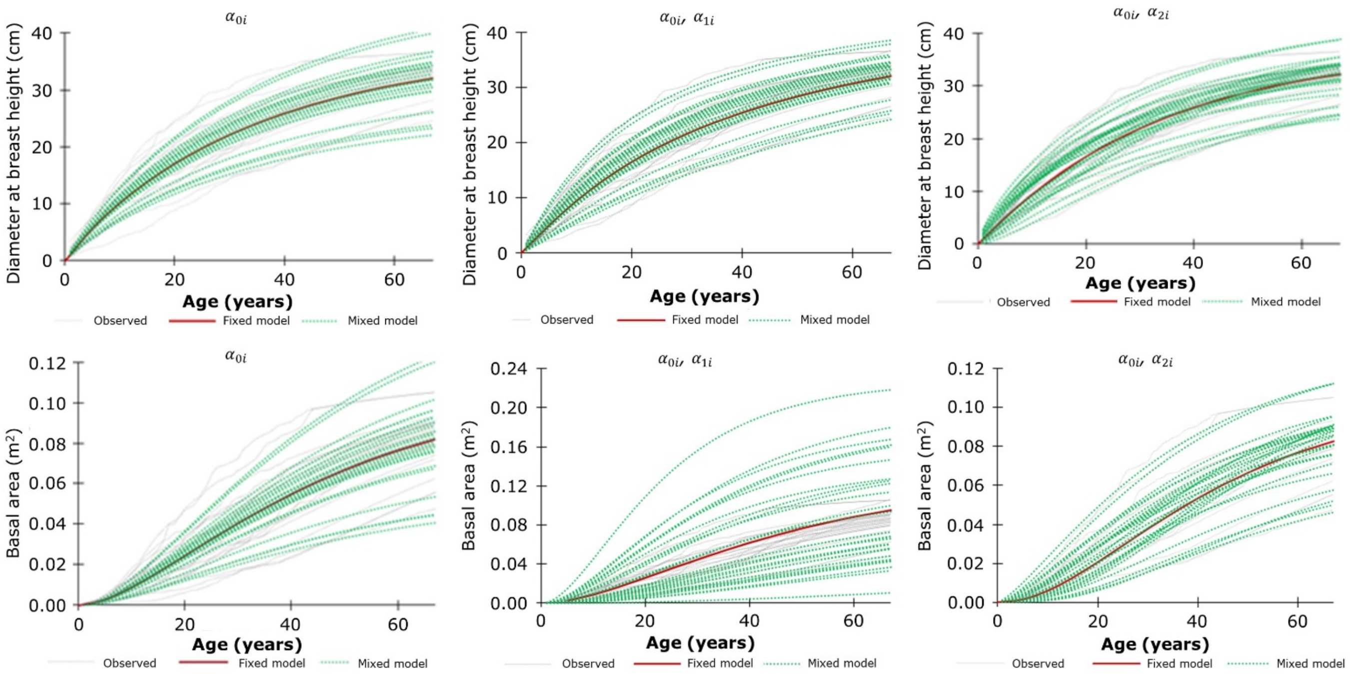


Figure 3. Behavior of diameter at breast height and basal area for each of the combinations of the mixed-effects model fit.

The results of the best fit for the diameter at breast height growth model coincide with those reported by Quiñonez-Barraza et al. (2015), who evaluated three Algebraic difference approach (ADA) and three Generalized algebraic difference (GADA) growth models to predict diameter at breast height growth as a function of age for six of the main commercial pine species in the state of *Durango*, with data from 44 302 growth cores collected from 15 609 forest inventory sites. Their results indicate that the Chapman-Richards polymorphic model (Richards, 1959) best predicted diameter growth for *Pinus lumholtzii* B. L. Rob. & Fernald with an R^2 of 0.99 and an *RMSE* of 0.63 cm, while for *Pinus ayacahuite* C. Ehrenb. *ex* Schltdl., a Verhulst-Logistic GADA model best predicted diameter at breast height growth with an R^2=0.99 and an *RMSE* of 0.64 cm.

Similar to the results of this study, Arteaga-Martínez (2000) identified the Chapman-Richards model as the best fitting, representing diameter growth in *Pinus pseudostrobus* with a fair statistical fit, but acceptable for practical use (R^2=0.8870).

The results of this study were also similar to those of Hernández et al. (2020), who fitted four diameter at breast height growth models for three *Pinus* species in the state of *Chihuahua*, Mexico. They used a sample of 82 trees and determined that the fits of the four models (Chapman-Richards, Hossfeld I, Schumacher, and Weibull) tested for basal area growth were good, with coefficients of determination of 0.96 to 0.98, respectively, and *RMSE* of 0.004 to 0.006.

Calvillo et al. (2005) described, in a similar way, the growth and increase in diameter at breast height and basal area of *Pinus herrerae* Martínez through stem analyses of 34 trees collected in the *Ciudad Hidalgo* region, state of *Michoacán*, Mexico. They tested the Chapman-Richards, Gompertz, Logistic, Schumacher, Weibull, and Exponential growth models. They found that the Gompertz model was the best model to describe diameter growth (*RMSE*=26.8730 and R^2=0.9697) and the Logistic model for basal area (*RMSE*=0.00037 and R^2=0.9374).

The MEM formulations made it possible to identify the variability of diameter at breast height and basal area growth for the tree source in each symmetric time series, and variation was assumed through fixed and random effects for combinations of parameters in the base Chapman-Richards model (Richards, 1959), and associated with MEM theory (Corral et al., 2019).

Normality tests for random parameters

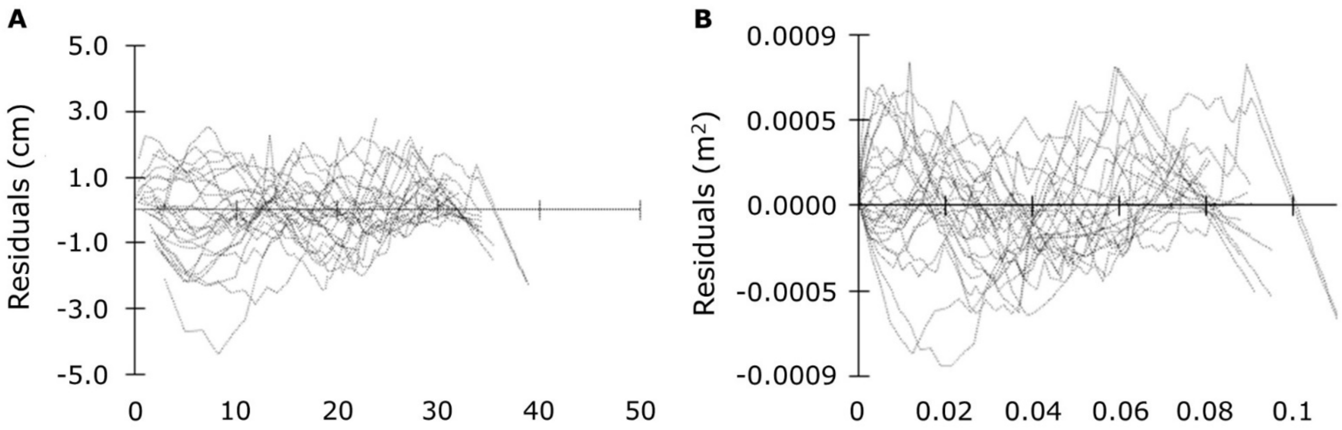
The normality tests performed on the random parameters of the diameter at breast height and basal area models, fitted as MEM, indicated that the random parameter representing the asymptote at the tree level for both study variables was normal, with values above the 1 % significance level. A value of 0.15 was calculated for the diameter at breast height model, while for the basal area model, it was 0.14 (Table 5). In both cases, the null hypothesis was not rejected, and it was assumed that the random parameters corresponding to α_{0i} for the 27 trees studied follow a theoretical normal distribution with a mean equal to zero and a known variance (Pinheiro et al., 2009). Furthermore, the time series of the stem analyses and the skewness in time allowed for satisfactory results.

Table 5. Shapiro-Wilks normality test for the diameter at breast height and basal area variables.

M	**Par.**	**Diameter at breast height**			**Basal area**			**Mean**
		SD	***W***	***p*-val**	***SD***	***W***	***p*-val**	
M10	α_{0i}	5.237	0.943	0.151	0.026	0.942	0.142	<0.00001
M11	α_{0i}	8.351	0.842	0.008	0.038	0.861	0.001	<0.00001
	α_{1i}	0.012	0.939	0.115	0.013	0.970	0.613	<0.00001
M12	α_{0i}	5.009	0.972	0.666	0.023	0.938	0.112	<0.00001
	α_{2i}	0.305	0.800	0.001	0.768	0.891	0.008	<0.00001

M = Model; Par. = Parameter; *SD* = Standard deviation; *W* = Shapiro-Wilk test; *p*-val = *p*-value.

Graphical analysis of the residuals against predicted values corroborated the performance of the M12 model in predicting growth in diameter at breast height and basal area (Figure 4). The trend in the residuals was found to be substantially symmetrical around the zero line. These residuals are presented as time series for each tree, assuming sampling site variability.



A = Diameter at breast height; B = Basal area.

Figure 4. Residual values *versus* predicted values of diameter at breast height and basal area growth with the inclusion of mixed effects.

Conclusions

All diameter and basal area growth models tested in this study showed good well performance; however, the model that suggested the best fitting statistics was the Chapman-Richards model. In turn, mixed-effects models (MEMs) improved predictions of diameter at breast height and basal area, using fixed and random parameters to reveal information about variability among trees for symmetrical time series from stem analyses. Random effects allowed for an implicit assessment of the sum of environmental variables in which the study species thrives.

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Conflicts of Interest

The authors declare no conflicts of interest. Gerónimo Quiñones- Barraza declares not having participated in any of the phases of the editorial process of the manuscript.

Contribution by author

Rosa Isela Delgado Espinoza: data analysis, model fitting, manuscript preparation and review; Francisco Cruz Cobos, Gerónimo Quiñones Barraza, Francisco Javier Hernández and Juan Abel Najera Luna: data analysis, manuscript preparation and review.

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