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Research article

Additive equations systems for inventories of the green weight of *Brahea dulcis* (Kunth) Mart. aboveground biomass

Sistemas de ecuaciones aditivas para inventarios del peso verde de la biomasa aérea de *Brahea dulcis* (Kunth) Mart.

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Abstract

The immature, folded leaf (spear) of the palm tree (*Brahea dulcis*) is an important non-timber forest product (NTFP) used by indigenous communities in the semi-arid regions of *Puebla*, Mexico. However, there is a lack of biometric tools to quantify the green weight of its biomass. Additive equations systems (AES) were developed to estimate the green weight of aboveground biomass by structural component and the total green weight of *B. dulcis* mature individual specimens. 42 specimens were collected using destructive sampling; for each standing individual, the stem diameter (D ; cm) was measured at a height of 20 cm above the ground; the total height (TH ; m) and the crown diameter (CD ; m) were also measured. The specimens were subsequently felled and separated into components (stem, petioles, green leaves, and spear), and their respective green weights (SW , PW , GLW , and SpW ; kg) were recorded. The total green weight (TGW) per individual was obtained by adding the weights of its components. Four AES were evaluated, using as a base model the potential allometric function $Y=\alpha \cdot X^{\beta}$; the fit was performed using the generalized method of moments. For the best AES selected ($R^2_{adj}=0.6919$ and $RMSE=0.8793$ kg for TGW), the predictor variables were the combination of $TH \cdot CD$, both of which are easy to measure. This AES will enable spear inventories to be carried out in compliance with the Mexican official regulations; furthermore, it is the first palm taxon, an important NTFP, to be documented in a semi-arid zone in specialized scientific literature.

Keywords: Crown diameter, allometric models, sugar palm, spear weight, non-timber forest product, additivity property.

Resumen

La hoja inmadura y plegada (velilla) de la palma (*Brahea dulcis*) es un importante producto forestal no maderable (PFNM) que aprovechan las comunidades indígenas de zonas semiáridas de Puebla, México. Sin embargo, se carece de herramientas biométricas que cuantifiquen el peso verde de su biomasa. Se desarrollaron sistemas de ecuaciones aditivas (SEA) que estiman el peso verde de la biomasa aérea por componente estructural y total de ejemplares individuales maduros de *B. dulcis*. Mediante un muestreo destructivo, se recolectaron 42 ejemplares; en cada individuo en pie, se midió el diámetro de tallo (DT; cm) a una altura de 20 cm sobre el suelo, la altura total (AT; m) y el diámetro de copa (DC; m). Posteriormente, se derribaron, seccionaron por componente (tallos, peciolos, hojas verdes y velilla), y se registró el respectivo peso verde (PT, PP, PHV y PV; kg); el peso verde total (PVT) por individuo se obtuvo sumando el peso de sus componentes. Se evaluaron cuatro SEA, el modelo base fue la función alométrica potencial $Y=\alpha \cdot X^\beta$; el ajuste se realizó con el método generalizado de momentos. Para el mejor SEA seleccionado ($R^2_{adj}=0.6919$ y $RCME=0.8793$ kg para el PVT) las variables predictoras fueron la combinación AT DC, ambas son fáciles de medir. Este SEA permitirá realizar inventarios de velilla para cumplir con la normatividad oficial mexicana; además, es el primero en su tipo que se documenta en la literatura científica especializada para un taxón de palma, importante PFNM, en una zona semiárida.

Palabras clave: Diámetro de copa, modelos alométricos, palma dulce, peso de velilla, producto forestal no maderable, propiedad de aditividad.

Introduction

Palms belong to the Arecaceae family and are of great cultural importance, as they have been used by humans for centuries as food (fruit and center of the stem) and to make many products that supplement both the family economy and self-consumption. Therefore, they are identified as a key non-timber forest product (NTFP) in several ecoregions around the world (Abdullah et al., 2020; Goodman et al., 2013; Pérez-Valladares et al., 2022).

Twenty genera and 100 species of palms whose management and use have been mainly empirical by different ethnic groups and indigenous communities have been identified in Mexico (Pulido-Silva et al., 2023).

Brahea dulcis (Kunth) Mart. (sweet palm, soyate palm, hat palm, or white palm) in particular is native to Mexico, Guatemala, Honduras, and Nicaragua (Barret et al., 2019). In Mexico, it is distributed in the states of *Chiapas, Coahuila, Colima, Guerrero,*

Guanajuato, Hidalgo, Jalisco, Morelos, Oaxaca, Puebla, Sinaloa, San Luis Potosí, Sonora, Tamaulipas and Veracruz (Pulido-Silva *et al.*, 2023). It grows in semi-arid environments, with dry climates and shallow limestone soils, on hills and at the foot of slopes, at altitudes ranging from 800 to 1 600 m. It can be a dominant species in tall palm groves, locally known as *soyacahuiteras*, and in low-growing groves called *manchonerias* (clumps). It also forms part of xerophytic scrubland and is associated with *Juniperus* sp. and *Quercus* sp. forests (Pérez-Valladares *et al.*, 2020; Rangel-Landa *et al.*, 2016).

Virtually all of the aerial parts of this palm (stem, petioles, leaves, bracts, fruits, and inflorescences) are used in over 100 different products (Pulido & Coronel-Ortega, 2015). The indigenous communities of the semi-arid region of the state of *Puebla* and other areas commercially exploit only the immature, folded leaves, known as spears (*velilla* or *cogollo*), which are used for making handicrafts, domestic utensils (basketry), religious and Christmas items, figurines, etc. (Aguilar *et al.*, 2005). Given the cultural and socioeconomic importance of this species for rural indigenous communities in *Puebla*, it is necessary to design programs for its technical management and sustainable use within the context of the prevailing regulatory and normative framework (Martínez-Pérez *et al.*, 2012; Pérez-Valladares *et al.*, 2020).

The use of palm leaves is regulated by the General Wildlife Law (Article 40) and its Regulations (Article 37), as well as by Mexican Official Standard NOM-006-SEMARNAT-1997 (López-Serrano *et al.*, 2021). For this purpose, it is necessary to prepare a technical study in the form of a management program (MP) for each forest property, which, among other aspects, indicates the amount of foliage to be extracted. The MP must be authorized by the Ministry of the Environment and Natural Resources (*Semarnat*). However, in the state of *Puebla*, a basic aspect that limits compliance with the technical principles for harvesting palm leaves is the lack of biometric tools to quantify the weight of the folded leaves (spears) of *B. dulcis*, which are essential because they allow inventories of the green weight of their biomass to

be made by establishing sampling sites, thereby determining their stocks and extraction possibilities.

The traditional way to estimate the weight of the leaves and spears is a destructive method whereby some specimens are weighed, and the value for all the palms in a given area is then extrapolated.

The logical strategy for estimating the green weight of aboveground biomass by structural component (stem, petioles, green leaves, and spear) and the total green weight of palms such as *B. dulcis* is to use allometric equations that form an additive equations system (AES), as is done for trees, in which, according to Bi et al. (2015), Cui et al. (2020), Fu et al. (2016) and Mohan et al. (2020), the additivity property is fully satisfied, meaning that the sum of the biomass weights of the components is equal to the total biomass weight. The practical implication of AES is that, subsequently, in determining the biomass of the taxon, it will suffice to apply a non-destructive sampling inventory method whereby the weight will be estimated based on easily measurable variables, such as, in the case of palms, the crown diameter or the total height of the specimen. This is similar to studies aimed at determining the biomass, carbon capture, and CO₂ sequestration of trees within the context of climate change in order to address it (Huy et al., 2023; Ordóñez-Prado et al., 2024).

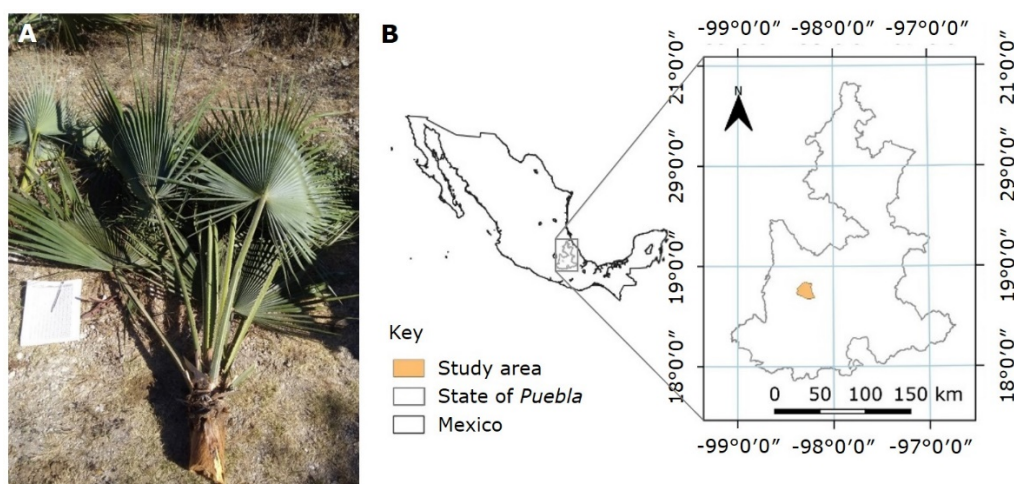
It should be noted that only for the states of *Guerrero* and *Oaxaca* have AES been generated to conduct inventories of the green aboveground biomass of *B. dulcis* by component and its total green weight (López-Serrano et al., 2021). In addition, it has also been determined that, among the tree palms of America, this species has the highest leaf production rate, with an interval of 11 to 15 leaves per individual⁻¹ year⁻¹ (Aguilar et al., 2005; Pulido & Coronel-Ortega, 2015).

Given the above scenario, the objective of this paper was set to develop allometric functions in the form of additive equations systems to estimate the weight of green aboveground biomass of commercially mature *Brahea dulcis* palm in *Puebla*, Mexico, by structural component (stem, petioles, leaves, and spears), as well as their total

green weight. The AES to be generated will form a set of biometric support tools for conducting inventories and harvesting this important NTFP.

Materials and Methods

In March and April 2024, a random sample of the aerial part of 42 specimens of the *Brahea dulcis* palm (Figure 1A) was collected in the community of *Teopantlán, Puebla*, Mexico (Figure 1B). Each specimen in the sample met the criteria of being well-formed, healthy and undamaged; in addition, their total height corresponded to the range considered to be mature and capable of producing commercial palm hearts, which is a minimum of 1.3 m. All specimens were collected from low-growing or dwarf palm groves (*manchoneras*), which are the sites where commercial extraction of spears takes place. The sample size was the minimum necessary and proved representative of the palm population in the study area. It was not possible to obtain more specimens because the community members who harvest spears have the ethnobiological and ecological view and knowledge to conserve their resources, and therefore prohibit the felling of *B. dulcis* individuals. For this reason, they allowed the removal of only 42 individuals to determine the green weight of the aboveground biomass through the generation of AES.



A = Collected *Brahea dulcis* (Kunth) Mart. specimen; B = Location of the study area in *Puebla*, Mexico.

Figure 1. Taxon and study area.

Each sample was measured using a model 283D Forestry Suppliers® 5 m long diameter tape, and the stem diameter (D) was measured in cm at a height of 20 cm from ground level. Subsequently, using a model PRO-55-MEB-R Pretul® 5.5 m long flexometer, the total height (TH) was measured in meters from ground level to the apex of the highest leaf. Also, the crown diameter was measured in the North-South and East-West directions using the same flexometer; then, the average crown diameter (CD) was calculated in m. In order to obtain the green weight corresponding to the aboveground biomass per component and the total green weight using a destructive method, the selected individuals were felled and divided into their components —stem, petioles, green leaves, and spears—, which were weighed independently using a Rhino® brand digital hanging scale with a capacity of 20 kg (kg) to record the weight of the stem (SW), petiole (PW), green leaves (GLW), and spear (SpW). The total green weight (TGW , kg) per specimen was obtained by adding up the green weight of the components.

The harvest took place during an atypical dry season, as the drought was more severe than the average recorded for the region. Because the main focus was to predict the fresh weight, which varies greatly depending on the season, a reference for the moisture content (*MC*) of the specimens was obtained by determining this parameter in 39 samples of spears; the *MC* interval was found to range between 14.7 and 60.0 % (33.6 % average).

An initial database (DB) was created in Excel using the measured variables, which was audited by inspecting graphs of the dependent variables (*SW*, *PW*, *GLW*, *SpW*, and *TGW*) vs. the predictor variables (*D*, *TH*, *CD*) to observe consistency and logical graphical behavior.

Next, a transformation process was carried out on the original predictor variables, which consisted of applying the power of 2 and the natural logarithm to each one according to Picard et al. (2015). Subsequently, both the original variables and the transformations were combined in a dual logical form. Furthermore, based on Han et al. (2020), independent variables were transformed by applying square root, exponential, and inverse functions, as well as their potential logical combinations. This made it possible to determine the best simple, combined, or transformed variable that explained the green weight of the aboveground biomass by structural component and the total green weight.

The final database with all the variables referred to, based on Picard et al. (2015), was used to adjust and evaluate the quality of fit of the potential nonlinear allometric model of the formula $Y = \alpha \cdot X^\beta$, where *Y* is the green weight of the aboveground biomass, *X* is the predictor variable, and α and β are regression parameters. This model served as the basis for developing and fitting different biomass additive equations systems (AES).

According to Fu et al. (2016) and Ordóñez-Prado et al. (2024), the general mathematical structure used as a basis for evaluating the fit of different biomass AES models is presented in expressions (1) to (5). The error term in each equation was

considered to have an additive effect (Chen et al., 2023), because being correlated in the AES results in more efficient and accurate estimates, since the standard errors are smaller, and, besides, the additivity property of the system is satisfied by summation.

$$SW = e^{\beta_0} \cdot X^{\beta_1} + \varepsilon_{1i} \quad (1)$$

$$PW = e^{\beta_2} \cdot X^{\beta_3} + \varepsilon_{2i} \quad (2)$$

$$GLW = e^{\beta_4} \cdot X^{\beta_5} + \varepsilon_{3i} \quad (3)$$

$$SpW = e^{\beta_6} \cdot X^{\beta_7} + \varepsilon_{4i} \quad (4)$$

$$TGW = e^{\beta_0} \cdot X^{\beta_1} + e^{\beta_2} \cdot X^{\beta_3} + e^{\beta_4} \cdot X^{\beta_5} + e^{\beta_6} \cdot X^{\beta_7} + \varepsilon_{0i} \quad (5)$$

Where:

SW , PW , GLW , SpW , and TGW = Green weight in kg of the aboveground biomass of the structural components stem, petiole, green leaves, and spear, and the total green weight, respectively

X = Simple, composite, or transformed predictor variable

β_i = Parameters to be estimated

ε_{ki} = Random error terms for the i^{th} equation

e = Exponential function applied to the parameter corresponding to the intercept

The parameters of the different AES were estimated using the generalized method of moments, which, according to Wang et al. (2018) and Xiong et al. (2023), is an appropriate technique when the sample size is small. Furthermore, these authors state that this technique is flexible and makes it possible to overcome the phenomenon of heteroscedasticity, as well as calculate efficient and consistent parameters, thereby obtaining robust standard errors and optimizing the root mean square error for each structural component and for the total green weight. The method also fully complies with the property of additivity.

To evaluate the quality of fit of the tested AES and select the best system, the significance of the estimated parameters ($p < 0.05$) and the goodness-of-fit statistics (Liu & Yen, 2021) Adjusted coefficient of determination (R^2_{adj}) and Root mean square error ($RMSE$) were considered. The heteroscedasticity was corrected by means of a graphical analysis of the distribution of the residuals against the TGW predictions.

A chart analysis of the observed vs. predicted values was performed to compare the graph trends generated by the resulting equation for the TGW (Xu et al., 2022), which reinforces the criteria for selecting the best AES, as it favors the one that best fits the trajectory described by the observed data.

The performance quality of the final AES selected was evaluated based on Cui et al. (2020) using the LOOCV (leave-one-out cross-validation) method. The same database (DB) was used for this validation. The statistics calculated were the Mean absolute error (MAE), the Mean prediction error (MPE), and the Mean percentage prediction error ($\%MPE$), in addition to the $RMSE$ and the R^2_{adj} .

To achieve biomass additivity, both the AES fitted and the validation process were performed simultaneously and compatibly using the Model procedure of the SAS 9.3 statistical package (SAS Institute Inc., 2011). The restriction imposed on each AES to comply with additivity was that the TGW of the palm specimen be equal to the sum of the green weight of the stem, petiole, green leaves, and spear components (Dong

et al., 2020); thus, the parameters and equations of the components are the same as those that make up the *TGW* expression.

Results and Discussion

Table 1 shows the basic descriptive statistics that quantitatively characterized the population of *B. dulcis* palm studied in terms of the analyzed predictor variables (*D*, *TH*, *CD*) and the green weight of the aboveground biomass by structural component and total green weight. The average percentage contribution per structural component to the *TGW* of biomass at the individual specimen level was 47.53 (± 3.87), 9.86 (± 0.91), 30.03 (± 3.16), and 12.58 % (± 1.57) for stem, petioles, green leaves, and spear, respectively.

Table 1. Descriptive statistics values for the analyzed variables of green weight of aboveground biomass for 42 specimens of *Brahea dulcis* (Kunth) Mart. palm specimens.

Variable	Minimum	Maximum	Mean	SD	CV	Variance
<i>D</i> (cm)	4.00	27.00	12.00	4.51	37.58	20.34
<i>TH</i> (m)	0.70	2.64	1.42	0.36	25.24	0.13
<i>CD</i> (m)	0.30	1.80	1.03	0.36	35.19	0.13
<i>SW</i> (kg)	0.14	4.19	1.01	0.84	82.49	0.70
<i>PW</i> (kg)	0.01	0.95	0.21	0.17	81.84	0.03
<i>GLW</i> (kg)	0.12	2.90	0.66	0.52	79.55	0.28
<i>SpW</i> (kg)	0.04	1.14	0.27	0.22	81.08	0.05
<i>TGW</i> (kg)	0.21	8.12	2.16	1.58	73.12	2.49

D = Stem diameter at a height of 20 cm from ground level; TH = Total height; CD = Crown diameter; SW , PW , GLW , SpW , and TGW = Green weight of stem biomass, petiole, green leaves, spear, and total green weight, respectively; SD = Standard deviation; CV = Coefficient of variation.

Of a total of 14 AES evaluated with a nonlinear structure, only four were found to be plausible and promising (Table 2) for estimating the green weight of biomass by structural component and the total green weight at the specimen level. In these AES, the predictor variables were the original variables. Table 3 shows the values of the parameters and adjustment statistics. Figure 2 shows the graphical behavior of the TGW estimates for the four AES.

Table 2. Mathematical structure of four promising additive equations systems (AES) for estimating the green weight of aboveground biomass by structural component and total for *Brahea dulcis* (Kunth) Mart. palm.

AES	Mathematical expressions
S1	$SW = e^{\beta_0} \cdot D^{\beta_1}$ $PW = e^{\beta_2} \cdot D^{\beta_3}$ $GLW = e^{\beta_4} \cdot D^{\beta_5}$ $SpW = e^{\beta_6} \cdot D^{\beta_7}$ $TGW = e^{\beta_0} \cdot D^{\beta_1} + e^{\beta_2} \cdot D^{\beta_3} + e^{\beta_4} \cdot D^{\beta_5} + e^{\beta_6} \cdot D^{\beta_7}$
S2	$SW = e^{\beta_0} \cdot (TH \cdot CD)^{\beta_1}$ $PW = e^{\beta_2} \cdot (TH \cdot CD)^{\beta_3}$ $GLW = e^{\beta_4} \cdot (TH \cdot CD)^{\beta_5}$ $SpW = e^{\beta_6} \cdot (TH \cdot CD)^{\beta_7}$ $TGW = e^{\beta_0} \cdot (TH \cdot CD)^{\beta_1} + e^{\beta_2} \cdot (TH \cdot CD)^{\beta_3} + e^{\beta_4} \cdot (TH \cdot CD)^{\beta_5} + e^{\beta_6} \cdot (TH \cdot CD)^{\beta_7}$
S3	$SW = e^{\beta_0} \cdot (D^2 \cdot TH)^{\beta_1}$ $PW = e^{\beta_2} \cdot (D^2 \cdot TH)^{\beta_3}$ $GLW = e^{\beta_4} \cdot (D^2 \cdot TH)^{\beta_5}$ $SpW = e^{\beta_6} \cdot (D^2 \cdot TH)^{\beta_7}$ $TGW = e^{\beta_0} \cdot (D^2 \cdot TH)^{\beta_1} + e^{\beta_2} \cdot (D^2 \cdot TH)^{\beta_3} + e^{\beta_4} \cdot (D^2 \cdot TH)^{\beta_5} + e^{\beta_6} \cdot (D^2 \cdot TH)^{\beta_7}$

$$\begin{aligned}
 S4 \quad & SW = e^{\beta_0} \cdot (D \cdot CD)^{\beta_1} \\
 & PW = e^{\beta_2} \cdot (D \cdot CD)^{\beta_3} \\
 & GLW = e^{\beta_4} \cdot (D \cdot CD)^{\beta_5} \\
 & SpW = e^{\beta_6} \cdot (D \cdot CD)^{\beta_7} \\
 & TGW = e^{\beta_0} \cdot (D \cdot CD)^{\beta_1} + e^{\beta_2} \cdot (D \cdot CD)^{\beta_3} + e^{\beta_4} \cdot (D \cdot CD)^{\beta_5} + e^{\beta_6} \cdot (D \cdot CD)^{\beta_7}
 \end{aligned}$$

AES = Additive equations system; *SW*, *PW*, *GLW*, *SpW*, and *TGW* = Weight of green biomass of stem, petiole, green leaves, spear, and total green weight, respectively;
D = Stem diameter; *TH* = Total height, *CD* = Crown diameter; *e* = Exponential function; β_i = Regression parameters.

Table 3. Parameter values estimated by the generalized method of moments for the four biomass additive equations systems and goodness-of-fit statistics by structural component and total aboveground biomass.

AES	Parameter	Estimator	SE	t-value	Pr> t	Eq	RMSE *	R ² _{adj} *
S1	β_0	-4.434470	0.5761	-7.70	<0.0001	SW	0.5827	0.5279
	β_1	1.755612	0.2254	7.79	<0.0001	PW	0.0994	0.6775
	β_2	-5.495300	0.6234	-8.82	<0.0001	GLW	0.3745	0.4925
	β_3	1.567078	0.2352	6.66	<0.0001	SpW	0.1647	0.4361
	β_4	-4.040990	0.7122	-5.67	<0.0001	TGW	0.9780	0.6193
	β_5	1.441822	0.2785	5.18	<0.0001			
	β_6	-5.154130	0.6054	-8.51	<0.0001			
S2	β_7	1.525077	0.2253	6.77	<0.0001			
	β_0	-0.506030	0.1138	-4.45	<0.0001	SW	0.5685	0.5527
	β_1	1.202718	0.1614	7.45	<0.0001	PW	0.0920	0.7241
	β_2	-2.169090	0.1280	-16.94	<0.0001	GLW	0.3361	0.5862
	β_3	1.350357	0.1675	8.06	<0.0001	SpW	0.1540	0.5122
	β_4	-0.933750	0.1299	-7.19	<0.0001	TGW	0.8793	0.6919
	β_5	1.172213	0.2176	5.39	<0.0001			
S3	β_6	-1.775110	0.1320	-13.45	<0.0001			
	β_7	1.114483	0.2123	5.25	<0.0001			
	β_0	-3.258860	0.4344	-7.50	<0.0001	SW	0.5318	0.5965
	β_1	0.603637	0.0772	7.82	<0.0001	PW	0.1097	0.6071

S4	β_2	-5.086410	0.3939	-12.91	<0.0001	GLW	0.3508	0.5532
	β_3	0.651036	0.0605	10.77	<0.0001	SpW	0.1425	0.5743
	β_4	-3.813300	0.4397	-8.67	<0.0001	TGW	0.8682	0.6972
	β_5	0.625400	0.0706	8.85	<0.0001			
	β_6	-4.746780	0.4031	-11.78	<0.0001			
	β_7	0.632040	0.0613	10.31	<0.0001			
	β_0	-2.756110	0.3285	-8.39	<0.0001	SW	0.4972	0.6472
	β_1	1.062817	0.1292	8.22	<0.0001	PW	0.0981	0.6858
	β_2	-4.361330	0.4317	-10.10	<0.0001	GLW	0.3471	0.5627
	β_3	1.079883	0.1552	6.96	<0.0001	SpW	0.1587	0.4724
	β_4	-2.880890	0.5111	-5.64	<0.0001	TGW	0.8211	0.7292
	β_5	0.952985	0.1944	4.90	<0.0001			
	β_6	-3.599960	0.5170	-6.96	<0.0001			
	β_7	0.888946	0.1990	4.47	<0.0001			

AES = Additive equations system; *SW*, *PW*, *GLW*, *SpW*, and *TGW* = Green weight of stem, petiole, green leaves, spear, and total green weight, respectively; *SE* = Standard error; Eq = Equation; *RMSE** = Root mean square error of each AES equation; R^2_{adj} * = Adjusted coefficient of determination for each AES equation; β_i = Regression parameters.

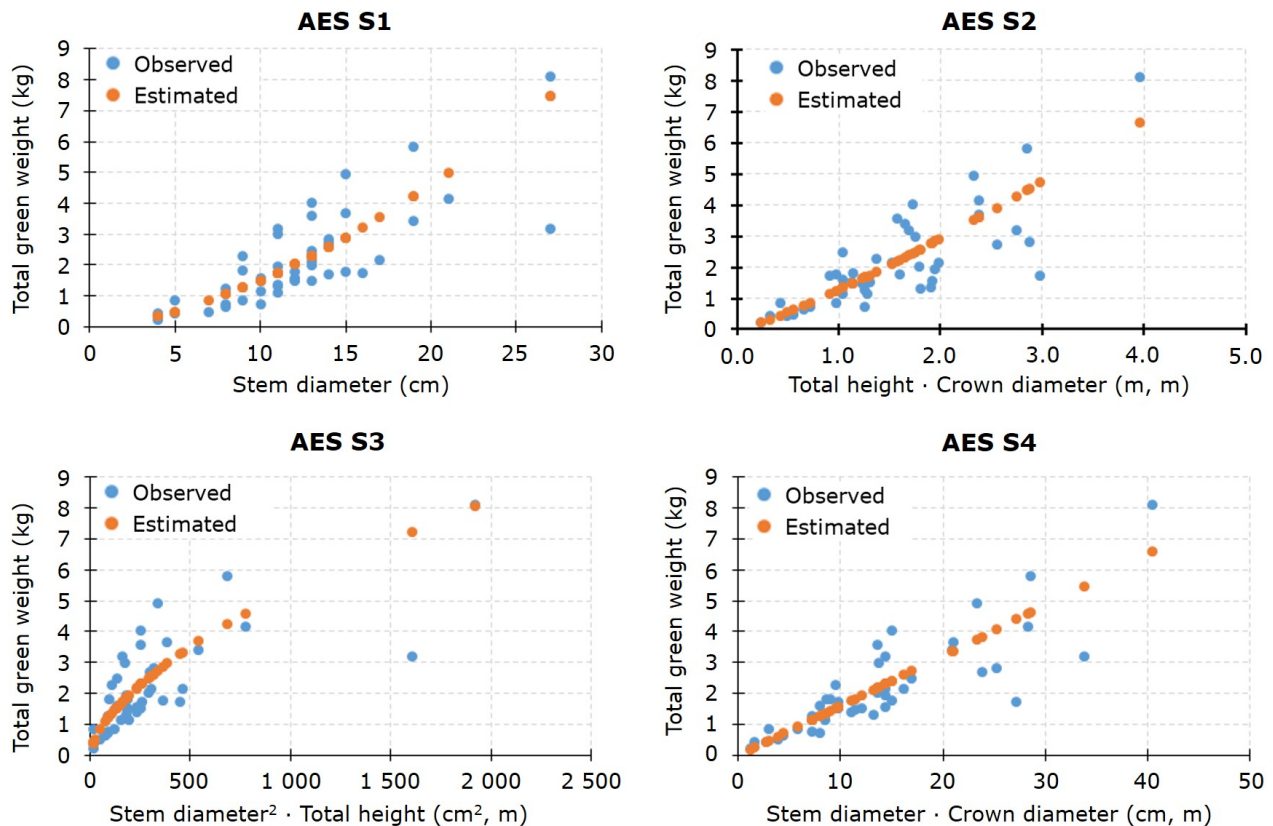
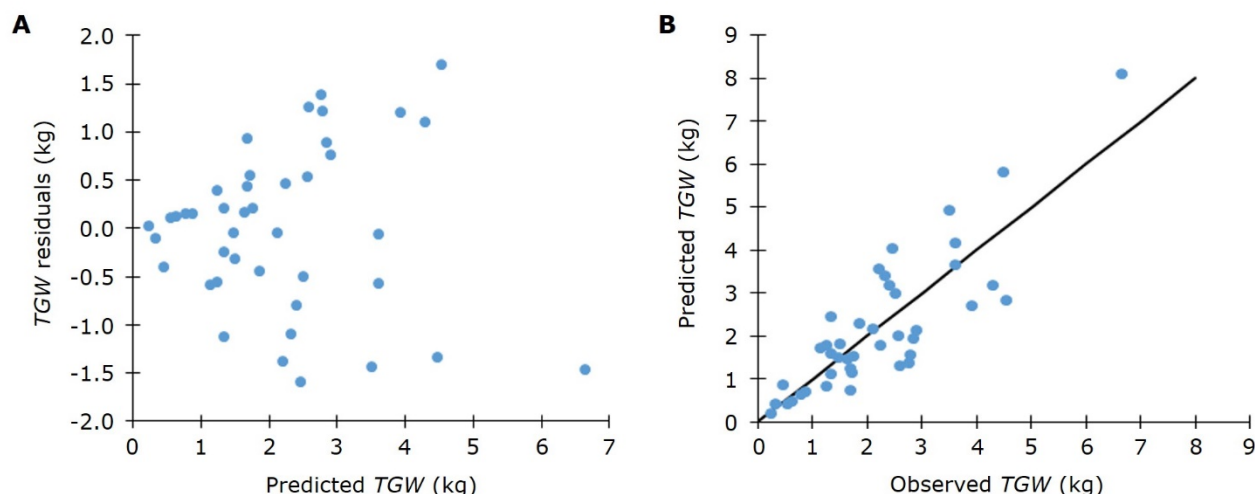


Figure 2. Graph behavior of estimates of total green aboveground biomass weight per specimen vs. those observed for the four selected additive biomass equations systems.

Considering the significance of the parameters, the goodness-of-fit statistics, the graph behavior of the estimates of each AES with respect to the observed values, and the relative ease of measuring the predictor variables per specimen in the field, AES S2 was selected. The correction of heteroscedasticity in this system using the GMM method was partial, because the residuals of the *TGW* compared to the predicted *TGW* values showed a slightly heteroscedastic trend (Figure 3A), which can also be seen in the scatter plot referring to predicted vs. estimated *TGW* values (Figure 3B). Table 4 summarizes the values of the statistics resulting from the LOOCV validation process applied to this AES.



A = TGW residuals vs. TGW predictions; B = TGW predictions vs. TGW observations.

Figure 3. Verification of the heteroscedasticity correction.

Table 4. Calculated values of the statistics applied to the LOOCV validation process of the AES S2.

Component	MAE	MPE	%MPE	RMSE	R^2_{adj}
Stem	0.4849	0.0693	6.69	0.5679	0.5529
Petiole	0.0761	0.0027	1.25	0.0919	0.7236
Green leaves	0.2859	0.0089	1.34	0.3354	0.5860
Spears	0.1236	0.0028	1.01	0.1537	0.5121
Total green weight	0.7861	0.0837	3.82	0.8792	0.6907

MAE = Mean absolute error; MPE = Mean prediction error; %MPE = Mean percentage prediction error; RMSE = Root mean square error; R^2_{adj} = Adjusted coefficient of determination by the number of parameters.

Based on the average values determined at $TH=1.4$ m and $CD=1.0$ m (Table 1), it could be assumed that *B. dulcis* is comparatively smaller in *Puebla* than in the state of *Guerrero*, where an average TH of 1.9 m and CD of 1.9 m are recorded (López-Serrano et al., 2021). With regard to the percentage contribution per structural

component to the green weight of the total aboveground biomass, the findings of this study are consistent with the trend observed by López-Serrano et al. (2021), according to whom the stem contributes the largest proportion (47.5 % in *Puebla* and 88.8 % in *Guerrero*), followed by green leaves (30.0 % in *Puebla* and 6.2 % in *Guerrero*), with the spear contributing 12.5 % in *Puebla* and 2.0 % in *Guerrero*.

The four AES evaluated showed highly significant parameters ($\alpha=0.05$) (Table 3), as well as efficient and biologically realistic behavior with respect to the observed data (Figure 2). The independent variable D had the lowest predictive power for TGW . According to Goodman et al. (2013), this is because palms are monocotyledonous and lack secondary growth, and therefore allometry with D as a predictor could be weak and can only improve when D is combined with another predictor variable such as TH or CD , as in the case of S3 and S4 AES, whose R^2_{adj} goes from 61.9 % when only D is used to 69.7 and 72.9 % when combined with TH or CA , respectively. However, it is difficult to measure the stem diameter (D) in this taxon because its growth and reproduction habit through shoots produces a high density of clumped palms in a small space, and, on the other hand, there are a large number of dry leaves attached to the stem, which must be removed in order to take measurements 20 cm above ground level. Consequently, it takes more time to measure each specimen when sampling for the purpose of estimating a biomass inventory.

AES S2 considers the combination $TH \cdot CD$ as a predictor variable, and therefore exhibits superior fit statistics for PW and GLW than AES S3 and S4 (Table 3). This combination of variables accounts for 51.2 % of the spear weight and 69.2 % of the TGW . The standard errors of the parameters for all biomass components for AES S2 are low, and the precision for TGW given by the $RMSE$ is similar to AES S3 and S4, at only 0.8 kg. Meanwhile, the values obtained for the validation statistics (Table 4) are conclusive in asserting that AES S2 has good predictive capacity, providing certainty for its practical and operational use. Thus, given that AES S2 exhibited: (1) Excellent values in the statistics calculated in the validation, (2) An adequate

graph behavior of the observed vs. estimated values for TGW (Figure 2), and that (3) *TH* and *CD* are easier and faster to measure than *D*, it is recommended for conducting green weight inventories of the aboveground biomass by structural component and the total green weight of commercial specimens of *B. dulcis* in the study area.

For *B. dulcis* palms growing in the state of Guerrero, López-Serrano et al. (2021) generated a linearized AES for the same purpose as the AES S2 in this study, with *TH* and *CD* as separate predictor variables and an $R^2_{adj}=76.1$ for *TGW*. They also applied a simultaneous adjustment to achieve green weight additivity, a property that, according to Huy et al. (2023) and Xu et al. (2022), allows biologically consistent estimates to be obtained without generating bias when scaling the final weight estimate at the surface level.

Equations to calculate the total aboveground biomass of tree palms in natural tropical forests or plantations have been developed that do not estimate the biomass contributed by each structural component and, therefore, do not generate biomass additive equations systems. Some of these palm species are: *Astrocaryum murumuru* Mart., *Attalea phalerata* Mart. ex Spreng., *Bactris gasipaes* Kunth, *Iriartea deltoidea* Ruiz & Pav., *Mauritia flexuosa* L. f., *Mauritiella aculeata* (Kunth) Burret, *Prestoea montana* (Graham) G. Nicholson, and *Socratea exorrhiza* (Mart.) H. Wendl. (Goodman et al., 2013), *Elaeis guineensis* Jacq. (Ramos-Escalante et al., 2018), *Areca catechu* L. (Das et al., 2021), *Oenocarpus bataua* Mart. and *Euterpe precatoria* Mart. (Falen et al., 2023) and *Raphia laurentii* De Wild. (Bocko et al., 2023).

In contrast, the additive allometric equations by structural component and total green weight developed in this study are the first of their kind to be documented in the specialized scientific literature specifically for a shrub-type palm taxon in a semi-arid ecological zone. Their main purpose is to quantify stocks in terms of the green weight of the aboveground biomass for the commercial use of the immature leaves (spear) as an NTFP. The AES generated with these equations will enable the estimation of the

green weight of the spear in order to comply with current federal forestry regulations and standards.

In future studies on additive equations for estimating the green weight of *Brahea dulcis*, the sample size must be larger and the collection must be carried out at different times of the year in order to achieve greater representativeness. Furthermore, it is important to avoid collecting specimens during periods of intense or prolonged drought. It is also necessary to determine the average moisture content and the respective moisture content range of the palm samples that are collected and processed.

Conclusions

Based on the evaluation of four nonlinear additive equations systems for estimating the green weight of the aboveground biomass by structural component (stem, petioles, leaves, and spear) and total for individual commercial specimens of *Brahea dulcis* from the state of *Puebla*, Mexico, the selected system: $TGW = e^{\beta_0} \cdot (TH \cdot CD)^{\beta_1} + e^{\beta_2} \cdot (TH \cdot CD)^{\beta_3} + e^{\beta_4} \cdot (TH \cdot CD)^{\beta_5} + e^{\beta_6} \cdot (TH \cdot CD)^{\beta_7}$ (RMSE of 0.879 kg in the TGW) it is parsimonious and uses total height and crown diameter as predictor variables, both expressed in meters and easy to measure. Its application will contribute to compliance with current regulations for conducting non-linear additive equation inventories of spears and the respective use of this important non-timber forest product from semi-arid areas.

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Conflict of interest

The authors declare that there is no conflict of interest. Juan Carlos Tamarit-Urias declares that he did not participate in the editorial process of the manuscript.

Contribution by author

Juan Carlos Tamarit-Urias: conceptualization and organization of the research, database creation, statistical analysis, drafting of the manuscript; Adrián Hernández-Ramos and Casimiro Ordóñez-Prado: preparation of images and graphs for the creation of figures, interpretation of results, and revision of the manuscript; Jonathan Hernández-Ramos and Enrique Buendía-Rodríguez: provision of specialized literature and revision of the manuscript. All authors participated in the editing of the manuscript.

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