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Article

Modelos dinámicos de índice de sitio para cuatro especies de pino en Oaxaca

Dynamic site index models for four pine species in *Oaxaca*

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Resumen

Los modelos de índice de sitio representan una herramienta silvícola muy importante para clasificar la productividad de los bosques. El objetivo de este trabajo fue desarrollar ecuaciones dinámicas de índice de sitio derivadas, mediante el método de Diferencias Algebraicas Generalizadas (GADA), para cuatro especies de pino en la Sierra Norte del estado de Oaxaca, México. Para el ajuste de las ecuaciones se utilizaron datos de análisis de tronco de árboles de *Pinus oaxacana*, *P. douglasiana*, *P. patula* y *P. pseudostrobus*. Los árboles se seleccionaron en rodales mixtos e irregulares, intentando cubrir las diferentes calidades de estación presentes en las áreas sujetas a manejo forestal en los bosques de la Unidad de Manejo Forestal Regional 2001. Los ajustes se realizaron con el método iterativo y una estructura de error autoregresiva de tiempo continuo de segundo orden (CAR2), para corregir la autocorrelación del término del error. Los resultados indicaron que la formulación GADA del modelo de *Bertalanffy-Richards* puede ser utilizada para describir con precisión el índice de sitio de las cuatro especies estudiadas. La función desarrollada es polimórfica, dinámica, invariable con la edad de referencia y tiene múltiples asíntotas. El modelo genera, para las cuatro especies, estimaciones compatibles de índice de sitio y de altura dominante.

Palabras clave: Altura dominante, curvas de índice de sitio, ecuación dinámica, edad de referencia, GADA, modelo de *Bertalanffy-Richards*.

Abstract

Site index models represent an important tool to classify forest productivity. The goal of this research was to develop dynamic site equations derived with the Generalized Algebraic Difference approach (GADA) for four pine species growing in the Northern *Sierra of Oaxaca*, Mexico. Data from stem analyses of *Pinus oaxacana*, *P. douglasiana*, *P. patula* and *P. pseudostrobus* trees were used to fit the equations. The trees were selected in mixed and irregular stands, trying to cover the different sites qualities present in the areas subject to forest management in the forests of the *Unidad de Manejo Forestal Regional* 2001(Regional Forest Management Unit 2001). The models were fitted through the iterative procedure, utilizing a second-order autoregressive error structure (CAR2) to correct for the serial correlation of term of error. The results indicated that the GADA formulation from the Bertalanffy-Richards equation can be used for accurately describing the site index for the four species studied. This function is polymorphic, dynamic and invariant with the reference age, with multiple asymptotes. It provides compatible site index and dominant height growth estimates for the four species.

Key words: Dominant height, site index curves, dynamic equation, reference age, GADA, *Bertalanffy-Richards* equation.

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Introduction

The productive capacity of a forest stand is defined by the tree species or group of species and by the biotic and abiotic factors that interact with the environment; therefore, the growth and production of timber in forest stands are determined by age distribution, the innate productive capacity supported by a given area, and its degree of utilization, as well as by the forestry treatments applied (Clutter *et al*., 1983; Torres and Magaña, 2001; Diéguez-Aranda *et al*., 2009).

The modern quantitative tools can be utilized to optimize the planning, follow-up and control of sustainable forest management, as they contribute to determine the rotation age, the silvicultural interventions and the timber harvest as accurately as possible. Thus, mathematical models make it possible to infer the biological behavior of individual trees or stands for the planning of forest management scenarios based on prediction and projection models (Quiñonez-Barraza *et al*., 2015).

The ratio of dominant height to age makes it possible to implicitly estimate the site index (SI), defined as the value of the dominant height of a given species at a reference base age (Clutter *et al.*, 1983); the latter is generally related to biological rotation ages in stands under forest management. A family of site index curves (also called site quality curves or productivity curves) is a set of curves that shows the dominant height growth pattern that will be followed through time by the stands of a particular species and a particular geographical area (Rennolls, 1978; Bengoa, 1999), each of which is associated to an Arabic number that identifies it (site index value), or, simply, to the SI value. In turn, the intermediate lines between quality curves delimit strips called quality classes.

The curves are anamorphic or polymorphic, and their equations are generated using the algebraic difference approach (ADA) (Baily and Clutter, 1974), or else, the families of curves with asymptotic polymorphism combining the growth rates with the potentiality of the site and obtained using the generalized algebraic difference approach (GADA) (Cieszewski and Bailey, 2000; Cieszewski, 2002; Cieszewski, 2003). This dynamic equations fitting method makes it possible for the variation in the instant growth rates of each curve and the potentiality of the site to be considered in a base equation; this is ideal for prescribing forestry treatments in stands, according to their productive potential (Santiago-García *et al.,* 2013).

In the forests of the *Unidad de Manejo Forestal Regional* 2001 (Regional Forest Management Unit 2001) (Umafor) in *Oaxaca*, Mexico, there are few site index equations for the species of the *Pinus* genus that can be used as biometric tools for planning and implementing sustainable forest management. The objective of this work was to develop dynamic site index models for *Pinus oaxacana* Mirov, *P. douglasiana* Martínez, *P. patula* Schiede ex Schltdl. & Cham. and *P. pseudostrobus* Lindl. in the Northern *Sierra of Oaxaca*, using the GADA methodology.

Materials and Methods

Study area

The study was carried out in the stands under forest management at Umafor 2001, in the northeast of the state of *Oaxaca*, Mexico, which cover an area of 195 397 ha. The study area is located at the coordinates 17°22’58” N and 96°22’29” W (Figure 1). The altitude of the sampling areas varies from 1 800 to 3 361 m. The annual temperature ranges between -3 °C and 28 °C, with a mean annual precipitation of 3 797 mm (Granados-Sánchez, 2009). The predominant vegetation type corresponds to pine-oak forests, with *Pinus oaxacana*, *P. douglasiana*, *P. patula*, *P. pseudostrobus*, *P. leiophylla* Schiede ex Schltdl. & Cham., *P. pringlei* Shaw., *P. rudis* Endl., *P. teocote* Schiede ex Schltdl. & Cham., *Quercus crassifolia* Bonpl., *Q. laurina* Bonpl. and *Q. rugosa* Née as the main dominant species.

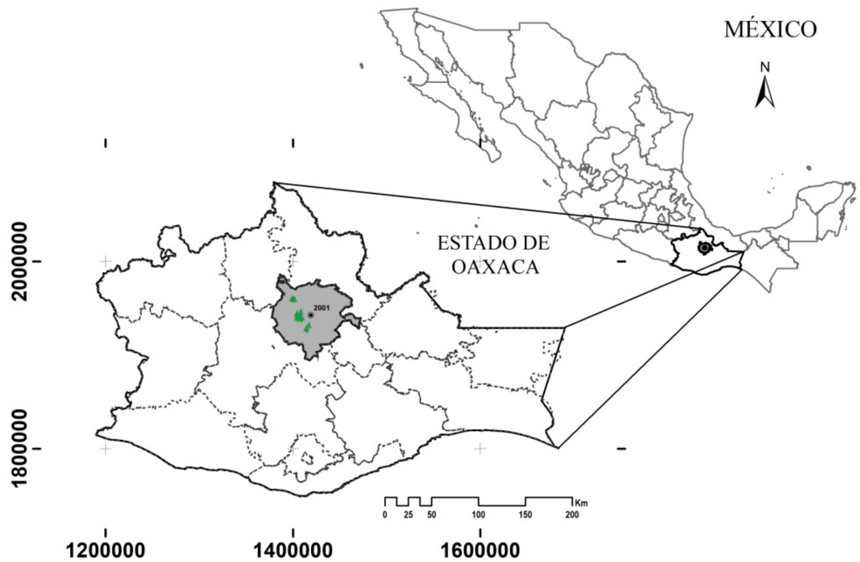


Figure 1. Geographic location of the study area.

Data

The dynamic SI and dominant height equations were fitted using a database of 206 tree stem analyses of 44 specimens of *Pinus oaxacana*, 45 of *P. douglasiana*, 58 of *P. patula*, and 59 of *P. pseudostrobus*. The individuals were felled at a stump height that ranged from 0.1 m to 0.25 m, so that the first section corresponded to the stump height, the second to a height of 1.3 m, and subsequently, every 2.54 m, up to the tip of the tree. The specimens were selected from mixed stands, and an attempt was made to cover the various site qualities, as well as the diameter categories present in the areas under forest management. Although the concept of site index is designed for coetaneous and pure stands (Daniel *et al.*, 1979), the dominant character for a tree implies that these individuals have little sensitivity to the effect of the density, the competition and the mix of species, as its height remains constant within a broad interval of densities and degrees of mingling, which reflects the productive potential of a particular site throughout its life, and therefore the concept has been extended to mixed pine stands (Corral *et al*., 2004; Vargas-Larreta, 2010; Castillo *et al.,* 2013).

Table 1 shows the descriptive statistics of the database; as it may be observed, the sample covers a broad interval of age-dominant height combinations, and therefore it is regarded as adequate for the fitting of the dynamic growth equations.

Table 1. Descriptive statistics of the age and dominant height data, by species, of the sample utilized to fitting the dynamic site index equations.

Code	Species	Trees	n	Age (years)				Dominant height (m)			
				Min.	Max.	Mean	SD.	Min.	Max.	Mean	SD
Po	*Pinus oaxacana* Mirov	44	563	2	66	21.8	13.7	0.1	36.1	13.5	9.2
Pd	*Pinus douglasiana* Martínez	45	638	2	126	36.2	24.1	0.1	38.8	15.3	9.9
Pp	*Pinus patula* Schiede ex Schltdl. & Cham.	58	804	2	87	27.3	19.1	0.1	43.7	15.4	10.6
Pps	*Pinus pseudostrobus* Lindl.	59	811	2	90	29.0	20.5	0.1	39.5	15.2	10.3
Total		206	2816	2	126	28.7	20.4	0.1	43.7	15.0	10.1

n = Number of pairs of dominant height - age; Min. = Minimum value; Max. = Maximum value; SD = Standard deviation.

The true heights of the sections were estimated using Carmean's method (1972) modified by Newberry (1991), as it has exhibited good results in other researches (Dyer and Bailey, 1987; Fabbio *et al*., 1994; Castillo *et al*., 2013). The method is based on two assumptions: i) between two sections, the tree grows at a constant rate, and ii) in average, the tree is cut at the growth core, at a one-year height (Dyer and Bailey, 1987; Amaral *et al*., 2010). The equations used to estimate the true height vary according to the tree section (equations 1-3).

Stump: $H_T = T * \left(\frac{H_2}{N_1 - N_2 + 0.5}\right)$ (1)

Log: $H_{T_{0+T}} = H_1 + \left[\left(\frac{H_2 + H_1}{2*(N_1 - N_2)}\right) + (T - 1) * \left(\frac{H_2 + H_1}{(N_1 - N_2)}\right)\right]$ (2)

Tip: $H_{T_{0+T}} = H_1 + \left(\frac{H_2 + H_1}{2*(N_1 - 0.5)}\right) + (T - 1) * \left(\frac{H_2 + H_1}{(N_1 - 0.5)}\right)$ (3)

Where:

H_1 and H_2 = Heights of the lower and upper sections of the log

N_1 and N_2 = Number of rings of the lower and upper sections of the log

N_0 = Age of the tree, *i.e.* number of rings of the stump (in stump $N_0 = N_1$)

T_0 = Age of the tree when it reached the height H_1; *i.e.* N_0 - N_1

T = Whole number from 1 to $N_1 - N_2$

Dynamic site index equations

The GADA method was used to fit three equations, which have been documented in various studies in order to describe the growth in dominant height and site index: Korf (4), Hossfeld (5) and Bertalanffy-Richards (6) (Cieszewski, 2002; Castillo *et al.*, 2013; Quiñonez-Barraza *et al.*, 2015; González *et al.*, 2016). Table 2 shows the expression of the base model, as well as their GADA formulation.

Table 2. Dynamic site index equations assessed in this study.

Base equation	Site parameters	Solution for X_0	GADA´s formulation	Equation
$H=a_1e^{-a_2t^{-a_3}}$	$a_1=e^X$ $a_2=(b_1+b_2)/X$ $a_3=b_3$	$X_0=\frac{1}{2}t_0^{-b_3}\left\{b_1+t_0^{-b_3}\ln(H_0)+\sqrt{4b_2t_0^{-b_3}+\left[-b_1-t_0^{-b_3}\ln(H_0)\right]^2}\right\}$	$H=e^{X_0}e^{\left[-\left(\frac{b_1+b_2}{X_0}\right)t^{-b_3}\right]}$	(4)
$H=\frac{a_1}{1+a_2t^{-a_3}}$	$a_1=b_1+X$ $a_2=b_2/X$ $a_3=b_3$	$X_0=\frac{1}{2}\left[H_0-b_1+\sqrt{(H_0-b_1)^2+4b_2H_0t_0^{-b_3}}\right]$	$H=\frac{b_1+X_0}{1+b_2/X_0t_0^{-b_3}}$	(5)
$H=a_1[1-e^{-a_2t}]^{a_3}$	$a_1=e^X$ $a_2=b_1$ $a_3=b_2+b_3/X$	$X_0=\frac{1}{2}\left[\ln H_0-b_2L_0+\sqrt{(\ln H_0-b_2L_0)^2-4b_3L_0}\right]$ Donde: $L_0=\ln\left[1-e^{-b_1t_0}\right]$	$H=H_0\left[\frac{1-e^{-b_1t_1}}{1-e^{-b_1t_0}}\right]^{b_2+b_3/X_0}$	(6)

H = Dominant height; t = Age in years; a_i = Parameters of the base equations; b_i = Global parameters for the GADA formulations; X_0 = Unobservable independent variable describing the productivity of the site with baseline values (t_0, H_0).

Description of the GADA method

The development of any dynamic equation using the GADA formulation considers: i) selecting a base equation and identifying the number of parameters that will be dependent on the productivity of the site; ii) the selected parameters are expressed as functions of the site quality defined by variable X_0 (unobservable, independent variable describing the productivity of the site, as a result of the sum of the ecological and climatic factors, management regimes, soil conditions and the new parameters; iii) the selected bi-dimensional base equation ($H=f(t)$) is expanded to a tri-dimensional site index equation ($H=f(t, X_0)$), and iv) the value of X_0 is solved in terms of the baseline conditions of the site, *i.e.* of the baseline dominant height values – age (t_0, H_0), so that the model can be implicitly defined and applied in practice ($H=f(t, t_0, H_0)$) (Cieszewski and Bailey, 2000; Cieszewski, 2001; Cieszewski, 2002; Cieszewski, 2003). In the course of the process, redundant parameters are often eliminated, and a model with an equal or lower number of parameters to that of the original base equation is obtained.

Fitting method and statistics for the comparison between models

The global and site-specific parameters were estimated using the nested iterative procedure (Tait *et al*., 1988; Cieszewski, 2003; Krumland and Eng, 2005), which is utilized with databases derived from tree stem analyses or permanent sampling plots. The models were fitted with a continuous-time autoregressive error structure (CAR_i), in order to correct potential autocorrelation problems of the term of error (Zimmerman *et al.*, 2001; Nord-Larsen, 2006; Crecente-Campo *et al*., 2009) and obtain unbiased, efficient parameter estimators (Parresol and Vissage, 1998).

$$\mathrm{H}_{ij} = f\left(H_j, t_i, t_i, \beta\right) + e_{ij} \qquad e_{ij} = d_1 p_1^{\,t_{ij}-t_{ij-1}} e_{ij-1} + \varepsilon_{ij} \qquad (7)$$

Where:

H_{ij} = Prediction of the height i using H_j (height j), t_i (age i)), and t_j (age $j \neq i$) as predictive variables

β = Vector of parameters to be estimated

e_{ij} = Error

$d_1 = 1$ for $j > 1$and zero when $j = 1$

ρ_1 = Autoregressive parameter of order 1, whose value is to be estimated

$t_{ij} - t_{ij-1}$ = Temporal difference separating the j_{th}observation from observation j_{th-1} in each tree stem analysis,$t_{ij} > t_{ij-1}$

The goodness-of-fit of the models was determined based on the results of quantitative analyses, such as the root mean square error (RMSE), the adjusted coefficient of determination (R^2_{adj}), the mean bias (e) and the residuals graphs for comparing the following aspects: i) the biological behavior of the predictions of the various fitted models; ii) the representation of the residuals *versus* the values predicted by the model, and iii) the representation of the residuals *versus* residuals with various lags, in order to verify the correction of the autocorrelation of the errors by modeling the error structure (Goelz and Burk, 1992; Sharma *et al*., 2011). The presence of autocorrelation in the utilized data was calculated with the Durbin-Watson statistic (Durbin and Watson, 1951). The expressions of these statistics are:

$$REMC = \sqrt{\frac{\sum_{i=1}^{n}(H_i - \hat{H}_i)^2}{n-p}} \quad (8)$$

$$R_{adj}^2 = 1 - \left[\frac{n-1}{n-p}\frac{\sum_{i=1}^{n}(H_i - H_i)^2}{\sum_{i=1}^{n}(H_i - \bar{H})^2}\right] \quad (9)$$

$$DW = \frac{\sum_{i=2}^{n}(e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^2} \quad (10)$$

Where:

H, $\hat{H}$ and $\bar{H}$ = Observed, predicted and mean values in pairs of height dominant data – age

n = Number of observations

p = Number of parameters of the model

e = Residual value of the fitted model

The simultaneous fitting of growth and error structure equations (CAR2) was carried out with the *MODEL* procedure of the SAS/ETS™ statistical package (SAS, 2004), which allows a dynamic updating of the residuals in the fitting procedure.

Results and Discussion

GADA equations generated without considering the error structure

The estimated parameters, the standard errors and the fitting statistics of the dynamic equations analyzed for *Pinus oaxacana*, *P. douglasiana*, *P. patula* and *P. pseudostrobus* were obtained for all the pairs of dominant height – age, with the fit of equations 4, 5 and 6, without considering the modeling of the error structure (Table 3). The results evidence autocorrelation problems of the residues, due to height measurements in the same tree (Figure 4).

Table 3. Estimated parameters and fitting statistics of the equations fitted without considering the modeling of the autocorrelation.

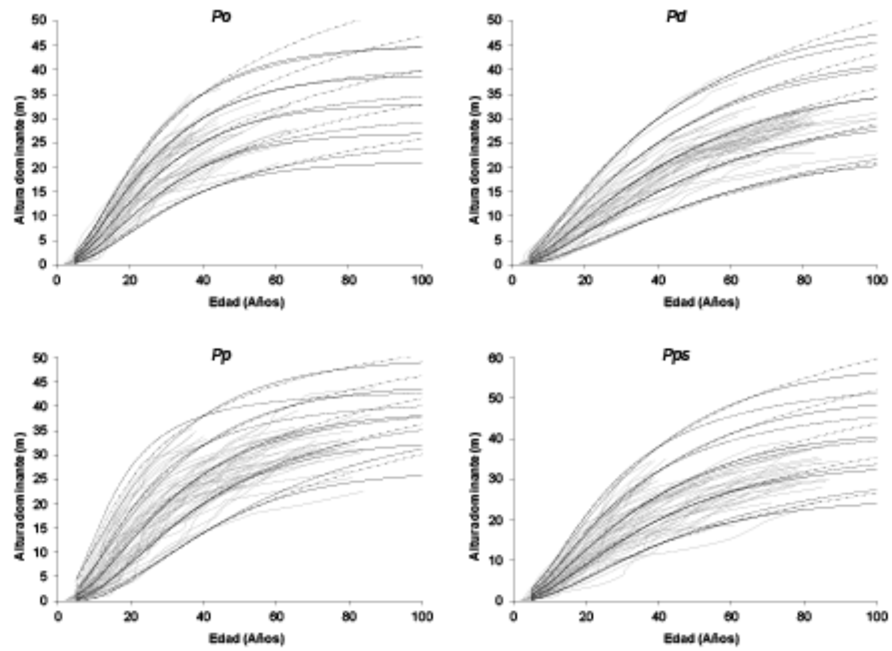
Species	Equation	Parameter	Estimated value	SE	T-value	P-value	RMSE (m)	R^2_{adj}	DW
Po	4	b_1	-9.9723	6.2560	-1.59	0.1115	1.23	0.98	0.73
		b_2	107.1878	27.0767	3.96	<0.0001			
		b_3	0.7720	0.0268	28.76	<0.0001			
	5	b_1	10.3192	4.3182	2.39	0.0172	1.22	0.98	0.70
		b_2	19 712.5600	3 295.9000	5.98	<0.0001			
		b_3	1.9817	0.0357	55.56	<0.0001			
	6	b_1	0.0546	0.0016	33.54	<0.0001	1.24	0.98	0.71
		b_2	-1.0411	0.5854	-1.78	0.0759			
		b_3	11.9310	2.1632	5.52	<0.0001			
Pd	4	b_1	-24.6624	4.8423	-5.09	<0.0001	1.24	0.98	0.62
		b_2	165.1131	21.2032	7.79	<0.0001			
		b_3	0.6067	0.0246	24.64	<0.0001			
	5	b_1	20.4518	2.3387	8.74	<0.0001	1.19	0.98	0.64
		b_2	11 664.3400	1 459.4000	7.99	<0.0001			
		b_3	1.6595	0.0292	56.74	<0.0001			
	6	b_1	0.0300	0.0010	29.98	<0.0001	1.18	0.99	0.65
		b_2	-1.8106	0.2727	-6.64	<0.0001			
		b_3	12.8909	1.0198	12.64	<0.0001			
Pp	4	b_1	-762.5240	289.6000	-2.63	0.0086	1.77	0.97	0.61

		b_2	3 227.2260	1 191.2000	2.71	0.0069			
		b_3	0.8459	0.0268	31.52	<0.0001			
		b_1	40.5627	0.8353	48.56	<0.0001			
	5	b_2	718.3985	491.1000	1.46	0.1439	1.74	0.97	0.59
		b_3	2.0036	0.0329	60.96	<0.0001			
		b_1	0.0447	0.0014	30.95	<0.0001			
	6	b_2	-9.4612	1.0574	-8.95	<0.0001	1.85	0.97	0.56
		b_3	42.7925	3.9189	10.92	<0.0001			
		b_1	-23.2364	5.8717	-3.96	<0.0001			
	4	b_2	163.9336	25.7222	6.37	<0.0001	1.49	0.98	0.61
		b_3	0.6937	0.0228	30.38	<0.0001			
		b_1	26.7838	2.0150	13.29	<0.0001			
Pps	5	b_2	10 535.2800	1 554.6000	6.78	<0.0001	1.47	0.98	0.62
		b_3	1.8045	0.0297	60.82	<0.0001			
		b_1	0.0368	0.0011	32.21	<0.0001			
	6	b_2	-0.7945	0.3176	-2.50	0.0126	1.51	0.98	0.60
		b_3	10.0448	1.2201	8.23	<0.0001			

SE = Standard error of the parameter estimator; RMSE = Root mean square error; R^2_{adj} = Adjusted coefficient of determination; DW = Durbin-Watson statistic; *Po*= *Pinus oaxacana*; *Pd* = *Pinus douglasiana*; *Pp* = *Pinus patula*; *Pps* = *Pinus pseudostrobus*.

The coefficients of determination (R^2_{adj}) adjusted by the number of parameters of the models of Korf, Hossfeld and Bertalanffy-Richards for the different species show that it is possible to account for more than 96 % of the variance observed in the dominant height growth, in terms of the of the age, with estimations of significant parameters for most combinations of equations and species ($p<0.01$). The mean errors in the three models range between 1.18 m (*Pd*) and 1.85 m (*Pp*) (Table 3).

In general, the three fitted equations show satisfactory fitting statistics for predicting the dominant height growth in terms of age, without expanding the terms of error in order to correct the autocorrelation. On the other hand, the graphic analysis shows a realistic biological behavior (similar predictions to the observed data). However, in *Pinus oaxacana* the parameter b_1 was not staticsally significant in the Korf (4) and Hossfeld (5) models; therefore, the null hypothesis that the parameter estimations are equal to zero is accepted; this scenario also occurred for the Bertalanffy-Rihards model (6) in parameter b_2. Parameter b_2 was not significant in the Hossfeld model (5) for *Pinus patula* Scheide ex. Schltdl. & Cham.; this situation occurred for the same parameter in the Bertalanffy-Richards model (6) for *Pinus pseudostrobus*, all at a significance level of 5 %.



Altura dominante = Dominant height; *Edad(años)* = Age(years).

Figure 2. Overlapping of SI curves generated using the Korf (discontinuous line), Hosffeld (dotted line) and Bertalanffy-Richards (continuous line) models.

The results show that the differences between the statistics of goodness-of-fit were minimal between the evaluated models; therefore, the selection of the most appropriate model was based on the graphic analysis of families of dominant height growth curves superimposed to the growth trajectories of the observed data; SI categories of 15 m to 35 m were considered for *Pinus oaxacana,* of 10 m to 30 m for *Pinus douglasiana*, with 5 m intervals, and of 14 m to 38 m for *Pinus patula* and *Pinus pseudostrobus*, with 6 m intervals, at a reference base age for all cases (Figure 2). This results from the fact that different models can exhibit the same goodness-of-fit or comparison statistics, but a different response to the trajectories of the observed data; thus, some underestimate these at the earlier ages and overestimate them at the later ages, or vice versa (Diéguez-Aranda *et al*., 2006).

The Betalanffy-Richards model (6) was selected as the most adequate for describing the growth trajectories of the trees included in the database, and in order to compare the growth estimations versus the same model, based on the modeling of the error structure, with CAR1 and CAR2 type continuous-time autoregressive error structures, as this model provides a slightly better description of the individual tendencies in dominant height growth, mainly at ages above 50 years (Figure 2).

The Bertalanffy-Richards equation, which considers a CAR2 structure, was able to model the growth tendencies of the experimental data for the studied species and describe variation patterns in the growth of the developed curves, as well as different site potentialities. This result is consistent with other studies which also recommend this model for the construction of site index curves for other species of commercial interest (Kiviste *et al*., 2002; Rodríguez-Carrillo *et al*., 2015; Castillo *et al*., 2013).

Error structure modeling

Table 4 shows the fitting statistics of the Bertalanffy-Richards model; this incorporates a continuous-time autoregressive model of order 2 (CAR2), which adequately corrected the autocorrelation of the database errors (DW value near 2).

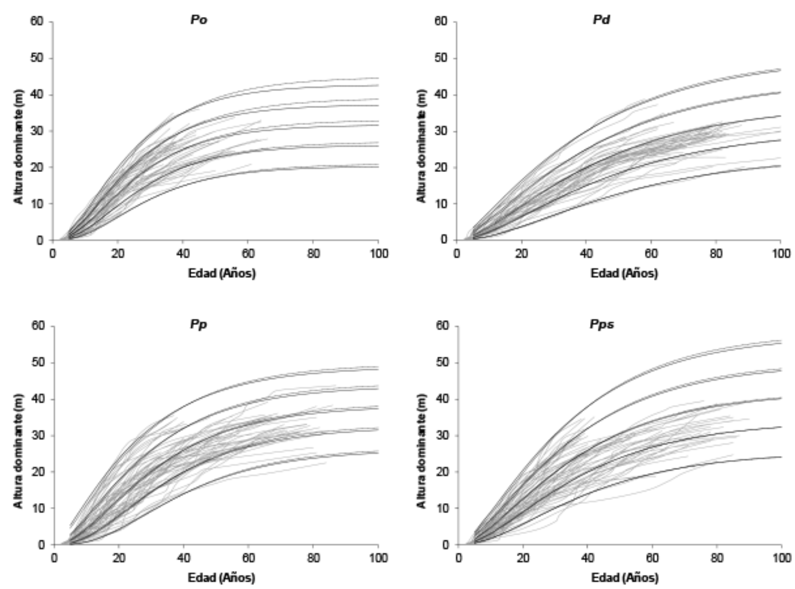
The average RMSE errors of model 6, with the error structure, ranged between 0.79 m (*Pd*) and 1.18 m (*Pp*) (Table 4), and accounted for more than 98 % of the variance in the dominant height growth. The continuous-time autoregressive model of order 2 (CAR2) ensured the independence of the residuals in the fitting process.

Table 4. Parameter estimations and goodness-of-fit statistics of the Bertalanffy-Richards equation, incorporating the continuous-time autoregressive model CAR2.

Equation	Species	Parameter	Estimated value	SE	T-value	P-value	RMSE (m)	R^2_{adj}	DW
		b_1	0.059830	0.00214	27.98	<0.0001	0.87	0.99	1.82
		b_2	-2.475350	0.94550	-2.62	0.0091			
6	*Po*	b_3	17.211490	3.42410	5.03	<0.0001			
		p_1	0.977894	0.00973	100.45	<0.0001			
		p_2	0.906721	0.01090	83.15	<0.0001			
		b_1	0.027946	0.00127	22.00	<0.0001	0.79	0.99	1.81
		b_2	-2.135910	0.40310	-5.30	<0.0001			
6	*Pd*	b_3	13.592790	1.49220	9.11	<0.0001			
		p_1	0.997808	0.00469	212.85	<0.0001			
		p_2	0.935772	0.00681	137.36	<0.0001			
		b_1	0.044495	0.00170	26.15	<0.0001	1.18	0.99	1.66
		b_2	-9.225630	1.31600	-7.01	<0.0001			
6	*Pp*	b_3	41.266710	4.85080	8.51	<0.0001			
		p_1	1.001470	0.00474	211.28	<0.0001			
		p_2	0.898239	0.00906	99.17	<0.0001			
		b_1	0.035901	0.00138	26.01	<0.0001	1.00	0.99	1.73
		b_2	-1.304410	0.41420	-3.15	0.0017			
6	*Pps*	b_3	11.565630	1.57430	7.35	<0.0001			
		p_1	0.978021	0.00671	145.79	<0.0001			
		p_2	0.917932	0.00803	114.28	<0.0001			

SE = Standard error of the parameter estimator; RMSE = Root mean square error, R^2_{adj} = Adjusted coefficient of determination; DW = Durbin-Watson statistic; *Po*= *Pinus oaxacana*; *Pd* = *Pinus douglasiana*; *Pp* = *Pinus patula*; *Pps* = *Pinus pseudostrobus*.

Figure 3 shows the SI statistics calculated with the Bertalanffy-Richards equation, without taking into account the error structure (dotted line), as well as those that do consider the CAR2 error structure (continuous line) at a base age of 40 years, overlapped on the experimental data utilized for fitting it. The SI curves cover various ages and describe the relationship of dominant height growth and age with a logical and biological behavior (complex polymorphism) at ages above 50 years (Cieszewski, 2003; Álvarez-González *et al*., 2010). This figure shows, graphically, that the studied species exhibit various dominant height growth patterns, indicating that stands with dominance of one of the four pine species require to be managed with different forestry schemes (Attis *et al.*, 2015).

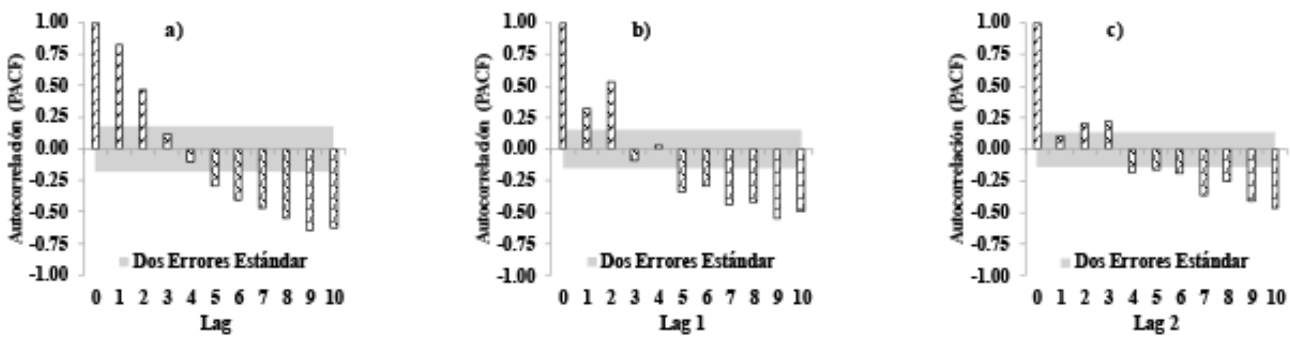


Altura dominante = Dominant height; *Edad(años)* = Age(years).

The curves were generated with the dynamic model of Bertalanffy-Richards, with (continuous line) and without (dotted line) correction of the autocorrelation using a CAR2 autoregressive structure.

Figure 3. Site index curves at the base age of 40 years for *Pinus oaxacana* Mirov (Po), *P. douglasiana* Martínez (Pd), *P. patula* Schiede ex Schltdl. & Cham. (Pp) and *P. pseudostrobus* Lindl. (Pps).

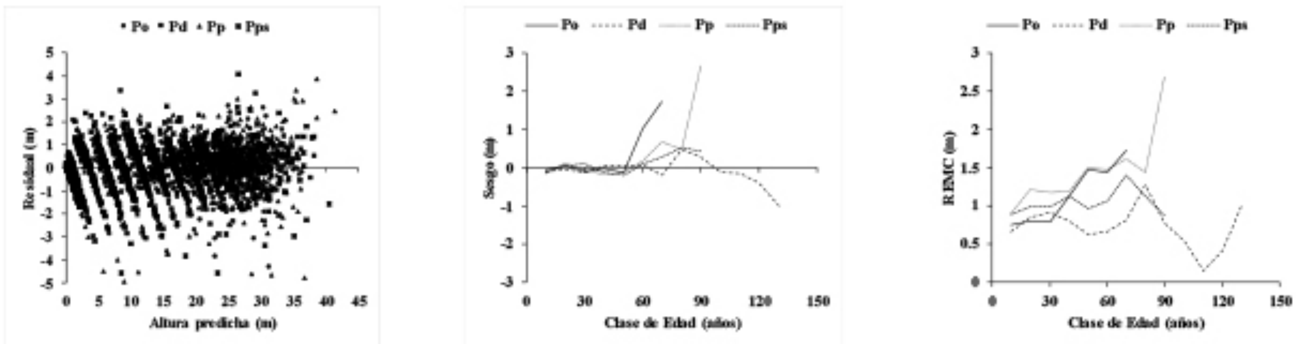
In the graphs below, the partial autocorrelation function (PACF) resulting from the fitting of the Bertalanffy-Richards equation with 10 lags of the residuals for each tree (Figure 4), is observed to change from a correlogram with a positive autocorrelation of order 2 to a correlogram without evidence of autocorrelation, estimated through the fit of the dynamic equation with a CAR2 type continuous-time autoregressive error structure; the results are similar to those registered by Vargas-Larreta *et al.* (2010), Vargas-Larreta (2013) and Quiñonez-Barraza *et al.* (2015) for forest tree species in various regions of Mexico.



Autocorrelación = Autocorelation; *Dos errores estándar* = Two standard errors.

Figure 4. Graphs of the Partial Autocorrelation Function of the Bertalanffy-Richards equation (Equation 6): a) fitting without correction of the autocorrelation, b) considering a CAR1 model, and c) considering a CAR2 model.

When correcting the autocorrelation, the random pattern of the residuals is defined around the zero line, with a homogeneous variance and without detecting a particular tendency (Figure 5). Also shown are the evolution of the bias and the RSME, as well as a considerable increase in the predicted dispersion and heights above the age of 50 years, due mainly to the lack of height-age data pairs for these age classes.



Residual = Residual; *Altura predicha* = Predicted height; *Sesgo* = Bias; *Clase de edad (años)* = Age category (years); REMC) RSME.

Figure 5. Residuals *vs* predicted heights and evolution of the bias and root mean square error (RSME) by age category in the Bertalanffy-Richards model, using the CAR2 autoregressive model.

Conclusions

The Bertalanffy-Richards dynamic equation is an adequate option for modeling the dominant height growth and site index for the four pine species studied at Umafor 2001. The obtained equations are polymorphic, with multiple asymptotes and invariants in relation to the reference base age; furthermore, they allow estimating the dominant height and the site index directly at any height or reference base age. The use of these equations will contribute to improve the estimation of the productive potential of those management units where *P. oaxacana, P. douglasiana, P. patula* and *P. pseudostrobus* are dominant, and their incorporation into forest management programs will allow selecting management regimes geared at increasing productivity.

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Conflict of interests

The authors declare to have the corresponding authorization for the use of the project data: Biometric system for the planning of the sustainable management of the ecosystems with timber potential in Mexico.

Contribution by author

Albert Castillo López: data analysis, model fitting, drafting of the manuscript; Wenceslao Santiago-García, Benedicto Vargas-Larreta, Gerónimo Quiñonez-Barraza, Raúl Solis-Moreno: review of the manuscript; José Javier Corral-Rivas: coordination of the reviews, statistical support and review of the manuscript.

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