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Article

Relación altura-diámetro para *Abies religiosa* Kunth Schltdl. & Cham. en el centro y sur de México

Height-diameter relation for *Abies religiosa* Kunth Schltdl. & Cham. in the center and south of the Mexico

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Resumen:

La altura total de los árboles es una variable importante en silvicultura y manejo forestal; sin embargo, su medición en campo es relativamente difícil y costosa. El objetivo de la presente investigación fue desarrollar una ecuación altura-diámetro que permita estimar de forma precisa la altura de árboles de *Abies religiosa* en el centro y sur de México. Para el ajuste de ecuaciones locales se utilizó un tamaño de muestra de 2 747 datos, que son suficientes para describir el comportamiento de las curvas sobre la variabilidad natural de la altura total de los individuos. Para la selección del mejor modelo se fijaron los siguientes criterios: coeficiente de determinación ajustada, sesgo, error medio; así como, el análisis gráfico y numérico de los residuales. La explicación de las variables independientes de cada modelo fue altamente aceptable, ya que todos fueron superiores (R^2_{adj}=0.96), además de presentar errores (REMC) por debajo de 0.44 metros y con sesgos cercanos a cero. Aunado a lo anterior, el modelo de *Bates* y *Watts* fue seleccionado mediante el análisis gráfico. Este tipo de ecuación es aplicable en las diferentes unidades de manejo forestal del país para trabajos de inventarios forestales; ya que reduce el tiempo, costo y, a su vez, minimizan los errores de campo.

Palabras clave: *Abies religiosa*, altura total, ecuaciones locales, manejo forestal, modelo *Bates* y *Watts*, relación altura-diámetro.

Abstract:

The total height of the trees is an important variable in forestry and forest management; however, its measurement in the field is difficult and expensive. The goal of this research was to develop a height-diameter equation that can accurately estimate the height of *Abies religiosa* trees in the center and south of the country. The local equations were adjusted using a sample size of 2 747 data, which are sufficient to describe the behavior of the curves on the natural variability of the total height of the trees. For the selection of the best model, the following criteria were established: adjusted coefficient of determination, bias, mean error, and graphical and numerical analyses of waste. The explanation of the independent variables of each model was highly acceptable, since they were all superior ($_{adj}R^2$ = 0.96), in addition to presenting errors (RMSE) below 0.44 meters and with near-zero biases. Combined with the above, the model of Bates and Watts was selected through graphical analysis. This type of equation can be applied to the different forest management units (*Umafores*) of the country for forest inventory purposes, since it reduces the time and cost while minimizing field errors.

Keywords: *Abies religiosa,* total height, local equations, forest management, model of Bates and Watts, height:diameter ratio.

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Introduction

Abies religiosa (Kunth) Schltdl. & Cham. is an economically and environmentally prominent species within its natural habitat. Therefore, it is important to have relevant information that may contribute to its management, growth and development (Vargas-Larreta *et al.*, 2017). Furthermore, various institutions have collaborated and shown interest in developing research in the different regions of Mexico in order to generate practical tools, such as the biometric models for supporting forestry technicians in improving the estimation of the timber yields of the forests under their management (Vargas-Larreta *et al.*, 2017), as well as for making better management decisions (Diamantopoulou and Özçelik, 2012; García-Cuevas *et al.*, 2016; Vargas-Larreta *et al.*, 2017).

The height:diameter ratio is used mainly for characterizing the vertical structure of the forest stands; *i.e.* it simulates forest growth (Burkhart and Strub, 1974; Wykoff *et al*., 1982; Larsen and Hann, 1987; Hernández-Ramos *et al*., 2018a), estimates the volume of the individual trees or of the tree mass, and determines the dominant height for the purpose of evaluating the seasonal quality (Huang *et al.*, 1992). It is equally important to know this ratio within other contexts, including forest biomass estimation, the simulation of the dynamics of the forest masses, and the analysis of the theoretical bases of tree growth (Canham *et al.*, 1994).

In most ecosystems of Mexico, heterogeneous masses can be observed due to the diversity of conditions that are prevalent in them, a fact that makes it difficult to use a single height:diameter equation for all species. However, equations can be adjusted for each tree of a particular species or stand (Arias, 2004) both locally and in general; the former estimate the height based exclusively on the normal diameter and certain variables of the stand (Diéguez-Aranda *et al.*, 2009). From this perspective, the purpose of the present study was to develop a height:diameter equation that will allow an accurate estimation of the total height of *Abies religiosa* trees in the central and southern regions of the country.

Materials and Methods

Study area

The study area contains information about 21 forest management units (*Umafores*) distributed in eight states: *Guerrero* (1203), *Puebla* (2101, 2105 y 2108), *Tlaxcala* (2901 y 2902), *Veracruz* (3004, 3012), *Michoacán* (1604, 1605, 1607 y 1608), *Jalisco* (1404, 1406 y 1410), *Hidalgo* (1303) and the State of Mexico (1503, 1507, 1508, 1509 y 1510) (Inegi, 2016). Table 1 describes some of the biophysical characteristics of the main regions (*Umafores*) and shows that the conditions for the development of *Abies religiosa* in them (soil, climate, and vegetation) are similar.

Table 1. Characterization of the study areas.

UMAFORES	**Altitude (msnm)**	**Climate**	**Type of soil**	**Type of vegetation**
1203	0 to 2 037	Warm subhumid with summer rains [A(w)], Semi-warm humid with abundant summer rains (ACm), Semi-warm subhumid with summer rains (ACw), Temperate subhumid with abundant summer rains [C(m)]	Arenosol (AR), Calcisol (CL), Cambisol (CM), Leptosol (LP), Luvisol (LV), Phaeozem (PH), Regosol (RG), others.	Grassland, Forest, Rainforest, Agricultural, Others
2101, 2105, 2108	130 to 2 829			
2901, 2902	2 200 to 2 738			
3004, 3012	0 to 2 420			
1604, 1605, 1607, 1608	10 to 2 595			
1404, 1406, 1410	29 to 2 347			
1303	137 to 2 712			
1503, 1507, 1508, 1509, 1510	1 126 to 2 808			

Source: Inegi (2016).

Sample size

Figure 1 shows the distribution of the diameter data used for estimating the height:diameter ratio and evidences compliance with the normal distribution assumption.

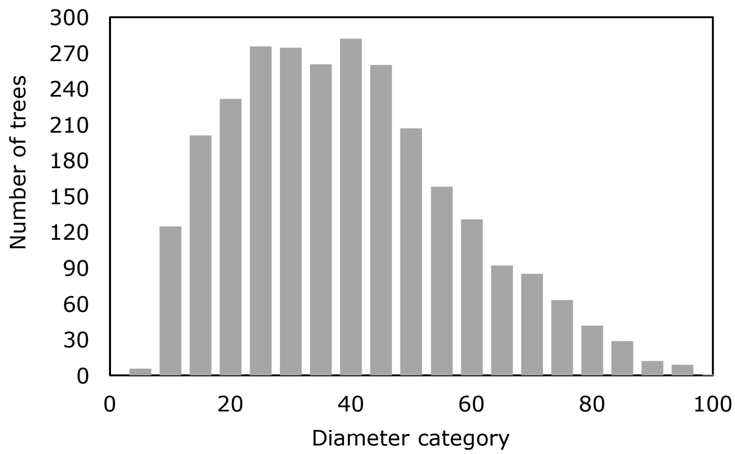

Figure 1. Distribution of the diameter categories.

Mensuration data were collected through random samplings of no more than 150 trees in harvesting areas, as well as in certain unauthorized areas. In the former case, destructive sampling was utilized; it consisted in felling, selecting and measuring the individuals chosen through targeted sampling that considered the various diameter categories. On the other hand, in the unauthorized areas, the measurements were staggered, and a Haglof Digitech Professionl laser caliper was used to measure the diameter at different heights. In all Umafores, the normal diameter was measured in the standing trees using a Forestry Suppliers, Inc. P.O.BOX 8397 diameter measuring tape and Haglof Mantax Blue calipers, measured at 1.30 above ground level; and the height was measured using a Haga®altimeter, except in the felled specimens, for which a Uline Accuc-Lock H-1766 flexometer was utilized.

Table 2. Summary of the stand variables

Num. of trees	Variables	Mean	Maximum	Minimum	Standard deviation
2 747	*Dn*	39.07	99	5.40	18.55
	At	24.64	52.45	5.30	9.05

Dn = Normal diameter; *At* = Total height.

Adjusted models

A first stage consisted in the fit of 10 non-linear local equations, which have been applied to describe the height:diameter ratio in several studies (Huang *et al*., 1992; Fang and Bailey, 1998; Peng, 1999), the difference between which lies in the number of parameters (Table 3). The equations were selected by means of a careful graphical analysis, since they all exhibited acceptable statistics.

Table 3. Local and general height:diameter ratio equations.

References	Expression	Equation
Local equations		
Bates and Watts (1980)	$At = 1.3 + b_0 * \left(\frac{Dn}{b_1 + Dn}\right)$	1
Stage (1975)	$At = 1.3 + b_0 * Dn^{b_1}$	2
Larson (1986)	$At = 10^{b_0} * Dn^{b_1}$	3
Wykoff *et al*. (1982)	$At = 1.3 + \exp\left(b_0 + \frac{b_1}{Dn + 1}\right)$	4
Richards (1959)	$At = 1.3 + b_0 * [1 - \exp(-b_1 * Dn)]^{b_2}$	5
Hossfeld (1822)	$At = 1.3 + \frac{Dn^2}{b_0 + b_1 * Dn + b_2 * Dn^2}$	6
Loetsch *et al* (1973)	$At = \frac{Dn^2}{b_0 + b_1 * Dn^2}$	7
Burkhart and Strub (1974)	$At = 1.3 + b_0 * \exp\left(\frac{b_1}{Dn}\right)$	8
Weibull (1951)	$At = 1.3 + b_0 * [1 - exp(-b_1 * Dn^{b_2})]$	9
Meyer (1940)	$At = 1.3 + b_0 * [1 - exp(-b_1 * Dn)]$	10

At = Total height (m); *Dn* = Diameter at the height of 1.3 m (cm); b_i = Parameters to be estimated (i = 0, 1, 2).

Adjustment method and selection of models

The adjusted models were non-linear, since this type of equations have a more consistent behavior from the biological point of view, which allows capturing the height:diameter ratios with a greater accuracy in terms of their predictive capacity (Huang *et al*., 1992; Diamantopoulou and Özçelik. 2012). The parameters were estimated using the least ordinary squares method (LOS), as it minimizes the errors of the parameters. In order to prevent the convergence of these to a local optimum, the values obtained by other authors in similar studies were used. The equations were fitted using the R software (R Core Team, 2017).

The goodness-of-fit analysis of the models was based on numerical and graphical comparisons. Based on the residuals obtained in the adjustment phase, the following statistics were estimated in order to compare and select the best models: the adjusted coefficient of determination ($_{adj}R^2$), for which a value of 1 is desirable, while the root mean square error (RMSE) tends to zero. The bias $(\bar{e})$ seeks to attain an average of the residuals that is equal to zero, in order to obtain a centered or unbiased estimator (Amat-Rodrigo, 2016).

The goodness-of-fit statistics are expressed as follows:

$$R^2_{adj} = 1 - (1 - R^2)\left[\frac{n-1}{n-p-1}\right]$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \widehat{Y}_i\right)^2}{n-p}}$$

$$\bar{e} = \frac{\sum_{i=1}^{n}\left(Y_i - \widehat{Y}_i\right)}{n}$$

Where:

p = Number of parameter to be estimated

n = Sample size

Y_i = Observed values

$\widehat{Y}_i$ = Predicted values

In addition, a graphical analysis of the residuals was carried out and checked against the predicted values for total height; this procedure is regarded as one of the most efficient ways of assessing the capacity of adjustment of a model (Diéguez-Aranda *et al*., 2005), since it detects potential systematic trends of the data, as well as of selecting weighting factors, in case this is required due to the presence of heteroscedasticity (Neter *et al.*, 1996).

Results and Discussion

Selected height-diameter equations

Table 4 summarizes the values of the estimated parameters and the model adjustment statistics. The estimators of the local parameters were different from zero (P<0.0001), with a 95 % confidence interval (Quiñones-Barraza *et al*., 2018). All the equations ensured an optimal confidence interval, as they exhibited errors of less than 1.667 m (RMSE) and adjusted coefficients of determination ($_{adj}R^2$) above 0.96. Likewise, all the models exhibited low biases: models 1, 2, 3, 5 and 9 had negative biases, *i.e.* they slightly overestimated the information; the rest of the equations produced positive biases, tending to an acceptable underestimation of total height, and therefore complying with the statistical property (Amat-Rodrigo, 2016). Equations 1 and 4 were selected based on this perspective.

Table 4. Estimated parameters and local goodness of fit statistics.

Parameters	Estimator	SE	T	P>t	Bias	RMSE	$_{adj}R^2$	Eq
Local equations								
b_0	119.8798	0.4630	258.41	<0.0001	-0.030	0.44	0.99	1
b_1	154.2599	0.7890	195.49	<0.0001				
b_0	1.3407	0.0070	176.24	<0.0001	-0.054	0.77	0.99	2
b_1	0.7854	0.0010	540.44	<0.0001				
b_0	0.216	0.0020	104.92	<0.0001	-0.040	0.68	0.99	3
b_1	0.7449	0.0010	611.36	<0.0001				
b_0	4.019	0.0030	1286.01	<0.0001	0.195	1.53	0.97	4
b_1	-31.1899	0.1330	-234.01	<0.0001				
b_0	63.1847	0.3380	186.48	<0.0001				
b_1	0.0135	0.0001	92.81	<0.0001	-0.001	0.39	0.99	5
b_2	1.0709	0.0030	284.57	<0.0001				
b_0	-0.0547	0.0630	-0.87	0.3868*				
b_1	1.3678	0.0030	439.32	<0.0001	0.004	0.40	0.99	6
b_2	-0.0028	0.00001	-153.17	<0.0001				
b_0	2.8883	0.0070	372.75	<0.0001	0.123	0.94	0.98	7
b_1	0.121	0.0001	716.11	<0.0001				
b_0	54.3213	0.1800	301.47	<0.0001	0.221	1.66	0.96	8
b_1	-29.3301	0.1370	-213.48	<0.0001				
b_0	70.5523	0.2130	331.30	<0.0001	-0.023	0.41	0.99	9
b_1	0.0107	0.00004	249.38	<0.0001				
b_0	61.7464	0.3740	164.95	<0.0001				
b_1	0.0104	0.00004	238.19	<0.0001	0.001	0.39	0.99	10
b_2	1.0568	0.0020	373.19	<0.0001				

*Note: the estimator of parameter b_0 is not significant ($p>0.05$)

SE = Standard Error of the parameter estimator; P>t = Value of the probability of Student's t-distribution; RMSE = Root mean square error; $_{adj}R^2$ =Adjusted coefficient of determination.

Comparison of the selected height-diameter equations

Figure 2 shows the adjustment curves of the local equations superimposed on the observed data, in order to explain the reasons for their selection. Although the statistics accompanied the equations fittingly, not all of them had a good graphical representation. Therefore, a detailed analysis of the charts was carried out to make a good selection. Equations 1 and 4 marked a visible difference (Rodríguez-Carrillo *et al.*, 2015); in the first case, height estimations are biologically plausible from the 5 cm category on. Furthermore, the predicted heights tend to zero, as the normal diameter diminishes (Diéguez-Aranda *et al*., 2005; Missanjo and Mwale, 2014), although this has no practical importance, since the inventories consider only trees with commercial measures (Puji, 2014).

The curve of model 4 did not exhibit a logical behavior for diameters under 15 cm, although in diameters of 20 to 70 cm it has an acceptable behavior, while in larger categories it loses consistency in the prediction. Besides, the residuals graph shows that they do not follow a normal distribution. It is therefore evident that goodness of fit is not the only option for selecting the best equations (Diéguez-Aranda *et al*., 2005).

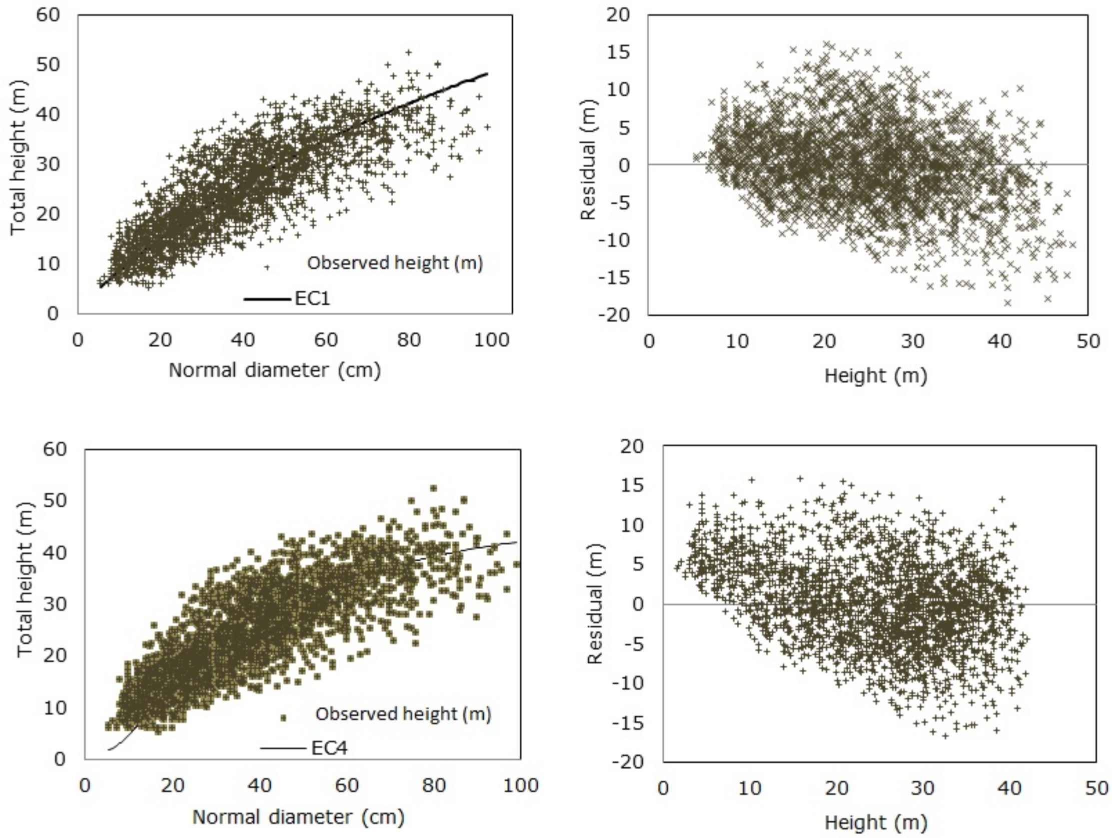

Figure 2. Height estimated with the equations (1 and 4) (left) and residuals (right).

Although the curve is correctly overlapped on the observed data, the residuals exhibit a marked margin of error in height. This type of problems frequently occur where the forest is very dense, as the trees are competing fully in growth and, consequently, they do not exhibit a noticeable increase in diameter. In addition, these equations have been designed to estimate larger diameters or diameters near the tip of the tree; *i.e*. when the tree reaches a height of 1.30 m, the diameter must be zero (Puji, 2014).

The results obtained by Hanus *et al.* (1999), López-Sánchez *et al.* (2003), Barrio-Anta *et al.* (2004) and Crecente-Campo *et al.* (2013) in various species support the information of the present study, since their residual graphs also exhibit a margin of error, which are probably trees from areas with a high density and competing for growth.

The local equation of Bates and Watts (1980) turned out to be as follows:

$$At = 1.3 + 119.8798 * \left(\frac{Dn}{154.2599 + Dn}\right)$$

Its application is appropriate in inventories with forest stands in different regions of the country or in future plantations. Its application requires only to measure a small sample of 25 trees and can subsequently be extrapolated to the areas of interest; this facilitates the field work (Arias, 2004; Diéguez-Aranda *et al*., 2009; Hernández-Ramos *et al*., 2018b).

Several studies point out that, despite the homogeneous characteristics of the forest areas, a local or simple height-diameter (h-d) model is usually insufficient to describe all the possible ratios in a stand because the height curves are not constant (Fang and Bailey, 1998; López-Sánchez *et al*., 2003; Castedo-Dorado *et al*., 2006; Trincado and Leal, 2006; Vargas-Larreta *et al*., 2009; Feldpausch *et al*., 2011; Ahmadi and Alavi, 2016). However, the results obtained show that its application is indeed possible, for sometimes a local equation (h-d) can be applied to one or several species in different regions of the world (Landsberg and Waring, 1997; Landsberg *et al.,* 2001; Puji, 2014; Arnoni-Costa, 2016), though these rare (Salas *et al.*, 2016).

On the other hand, López-Villegas *et al.* (2017) point out better adjustments with the local equations in various studied species, as the stands or management units have a certain degree of homogeneity and the data implicitly consider the quality of the site; this is not the case with global equations, which generate a greater variability and therefore require a slighter adjustment.

Conclusions

Adjusted equations exhibit no differences in goodness of fit; for this reason, a graphical analysis was carried out to select the appropriate equation for the estimation of heights. The proposed model can be utilized to describe the height:diameter ratio en forest inventories of the various forest management units (*Umafores*) in the country; this will facilitate the work by reducing the cost and time and therefore minimizing errors in the data collection.

Consequently, the model can be added to the equations of growth in order to estimate the existing timber volume. This is the first equation of this kind ever developed for *Abies religiosa* in different regions of the country, as this model type is not included in the national biometric system.

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Conflict of interests

The authors declare no conflict of interests.

Contribution by author

Juan Carlos Guzmán Santiago: planning, in-field data collection, analysis of the information, and drafting of the manuscript; Oscar Alberto Aguirre Calderón, Marco Aurelio González Tagle, Eduardo Javier Treviño Garza and Javier Jiménez Pérez: planning, analysis of the information, and revision of the document; Benedicto Vargas Larreta: statistical analysis of the information; Héctor Manuel De los Santos Posadas: planning of the research and revision of the manuscript.

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